Published by IOP Publishing for SISSA
Received: *October 22, 2008*
Accepted: *November 18, 2008*
Published: *December 9, 2008*

# The canonical transformation and massive CSW vertices for MHV-SQCD

**Tim R. Morris and Zhiguang Xiao**

*School of Physics and Astronomy, University of Southampton, Highfield, Southampton, SO17 1BJ, U.K.*
*E-mail:* T.R.Morris@soton.ac.uk, z.g.xiao@phys.soton.ac.uk

Abstract: The similarity of massive CSW scalar vertices and quark vertices can be understood using a kind of light-cone SUSY transformation presented in this paper. We also show that the canonical transformation generating the MHV-SQCD Lagrangian, can be fixed by applying this light-cone SUSY transformation to the canonical transformation for MHV-QCD obtained in paper arxiv:0805.0239. Most of the massive CSW vertices for SQCD can also be pinned down in this way.

Keywords: Supersymmetric gauge theory, Gauge Symmetry.



JHEP12(2008)028

## Contents


**1. Introduction** **1**

**2. Preliminary** **3**
- 2.1 Light-cone co-ordinates 3
- 2.2 Spinor conventions 3
- 2.3 SQCD Lagrangian and component fields 4

**3. Light-cone SQCD Lagrangian and light-cone SUSY transformations** **5**

**4. Canonical transformations for the MHV-SQCD** **7**

**5. The massive CSW vertices for SQCD** **10**

**6. Conclusion and discussion** **14**

**A. Light-cone Lagrangian for SQCD** **15**

**B. Summary of the canonical transformation for MHV-QCD** **17**

**C. Summary of the canonical transformation for MHV-SQCD** **18**

**D. Summary of the massive CSW vertices for MHV-QCD** **20**




## 1. Introduction

The striking simplicity of MHV amplitudes has been an inspiration for many recent developments in gauge theory. It was first proposed by Parke and Taylor [1] and then proved using recursion methods by Berends and Giele [2]. After about fifteen years, inspired by twistor string theory, Cachazo, Svrček and Witten discovered that tree-level amplitudes with arbitrary helicity configuration can be constructed using MHV amplitudes continued off shell in a particular way and connected using scalar propagators [3]. This proposal reduces the complexity of the amplitude computation dramatically. CSW rules were then extended to include quarks and to supersymmetric theories [4 − 7], and also were extended to include Higgs [8, 9] and electroweak gauge bosons [10]. There are also some applications of CSW rules in one loop calculations (such as in [11 − 17]).

As well as the proof given in [18], a particularly direct proof of these rules was found in [19] by generalising the idea from BCFW recursion relations [20]. But despite a preliminary attempt to derive the MHV Lagrangian [21], a full and clear understanding of the origin of CSW rules from quantum field theory was not presented until Mansfield's paper [22]. In his paper, Mansfield proposed a framework for deriving the MHV Lagrangian using a certain canonical transformation applied to the light-cone Lagrangian: CSW rules then followed from the vertices in this MHV Lagrangian. The concrete canonical transformation was obtained in [23] and later was extended in [24] to give a prescription for dimensional regularization of the MHV Lagrangian. It was also made clear that the previously missing pieces of amplitudes in the CSW prescription can be recovered by so called 'completion vertices', which come from the field transformation. Another approach to obtain the MHV Lagrangian is to use the twistor Yang-Mills theory [25, 26] by formulating the gauge theory on twistor space and then fixing a particular gauge.

In [27, 28], a canonical transformation was used to generate the massive CSW vertices involving massive scalars, and it was shown that these can also be obtained from the twistor Yang-Mills approach. A full canonical transformation for QCD including a quark field was constructed in a recent paper [29], in which CSW vertices for massive quarks are also presented. The same results have been reproduced from the twistor Yang-Mills approach [30] (although formulated in a different convention).

The similarity between the massive CSW scalar vertices in [27, 28] and massive quark vertices in [29] suggests a supersymmetric relation between these two vertices. In [31], the author uses a massive version of the supersymmetric Ward identity (massive SWI) [32] to understand this similarity. Since massive SWI can only be used on amplitudes, one should choose a suitable reference momentum of the massive quark to make the amplitudes involving a massive quark-antiquark pair be proportional only to the corresponding massive CSW vertices. Then the massive SWI can be used to relate them to amplitudes involving massive scalars, hence relating these two kinds of CSW vertices. Since the derivation of the massive SWI makes use of the explicit solution of massive Dirac equation, it also depends on the reference momenta of the solution. However, at the level of the MHV Lagrangian, one would not expect that a supersymmetric relation between vertices should depend on the on-shell solution of the Dirac equation. In the present paper, a kind of light-cone supersymmetric transformation at the Lagrangian level is presented to relate these two kinds of vertices directly. It is based on the observation that after the gauge fixing and integrating out of the non-dynamical fields, there is some supersymmetry left in the Lagrangian. Moreover, the complete canonical transformation for the MHV Lagrangian for supersymmetric QCD (MHV-SQCD) can be obtained by using this supersymmetry transformation on the results already obtained for MHV-QCD in [29]. Besides the relation between massive CSW vertices for massive quarks and scalars, other relations among massive CSW vertices for MHV-SQCD can also be obtained in this way. As a result, most massive CSW vertices can be fixed using these relations and the results from MHV-QCD. Another kind of light-cone supersymmetry is used in [33] in discussing MHV Lagrangian for $\mathcal{N} = 4$ gauge theory.

The paper is organised as follows: In section 2, we explain the conventions and notation used in this paper, which is a little different from [29]. In section 3, we derive the light-cone Lagrangian and provide the light-cone SUSY transformation. In section 4, we obtain all of the canonical transformation for MHV-SQCD using the light-cone SUSY transformation and the canonical transformation for MHV-QCD. In section 5, we derive the massive CSW



JHEP12(2008)028

vertices for MHV-SQCD. Section 6 contains the conclusions and discussion. Since we will use the canonical transformation and massive CSW vertices for MHV-QCD from [29] in our derivation, we summarise them in appendix B and D. In appendix A, we give the full light-cone Lagrangian for SQCD. In appendix C we summarise our results for the canonical transformation for SQCD.

## 2. Preliminary

### 2.1 Light-cone co-ordinates

The light-cone co-ordinates are defined as:

$$x^0 = \frac{1}{\sqrt{2}}(t - x^3), \quad x^{\bar{0}} = \frac{1}{\sqrt{2}}(t + x^3), \quad z = \frac{1}{\sqrt{2}}(x^1 + ix^2), \quad \bar{z} = \frac{1}{\sqrt{2}}(x^1 - ix^2). \tag{2.1}$$

We employ a compact notation writing $(p_0, p_{\bar{0}}, p_z, p_{\bar{z}}) \equiv (\check{p}, \hat{p}, \tilde{p}, \bar{p})$ and for momenta labelled by a number we write that number with decorations to denote the components of the momentum: $(\check{p}_n, \hat{p}_n, \tilde{p}_n, \bar{p}_n) \equiv (\check{n}, \hat{n}, \tilde{n}, \bar{n})$. For component $\tilde{p}$, we will omit the tilde in case it causes no confusion. In this notation, the Lorentz invariant reads

$$A \cdot B = \hat{A}\check{B} + \check{A}\hat{B} - A\bar{B} - \bar{A}B\,. \tag{2.2}$$

We also make extensive use of the bilinears:

$$(i\,j) = \hat{k}_i\tilde{k}_j - \hat{k}_j\tilde{k}_i\,, \quad \{i\,j\} = \hat{k}_i\bar{k}_j - \hat{k}_j\bar{k}_i\,. \tag{2.3}$$

### 2.2 Spinor conventions

We use the Weyl representation of the Dirac matrices:

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu_{\alpha\dot{\alpha}} \\ \bar{\sigma}^{\mu,\dot{\alpha}\alpha} & 0 \end{pmatrix}, \tag{2.4}$$

$$\sigma^\mu_{\alpha\dot{\alpha}} = (1, \vec{\sigma}^i)\,, \quad \bar{\sigma}^{\mu,\dot{\alpha}\alpha} = (1, -\vec{\sigma}^i)\,, \quad \sigma^{\mu\nu} = \frac{1}{4}(\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu)\,. \tag{2.5}$$

The massless Weyl spinors are solutions of Dirac equation $p_\mu\sigma^\mu_{\alpha\dot{\alpha}}\bar{\chi}^{\dot{\alpha}} = 0$ and $p_\mu\bar{\sigma}^{\mu,\dot{\alpha}\alpha}\chi_\alpha = 0$:

$$\chi_\alpha = 2^{1/4}(\sqrt{\hat{p}}, \bar{p}/\sqrt{\hat{p}})^{\mathrm{T}}\,, \qquad \bar{\chi}^{\dot{\alpha}} = 2^{1/4}(p/\sqrt{\hat{p}}, -\sqrt{\hat{p}})^{\mathrm{T}} \tag{2.6}$$

$$\chi^\alpha = 2^{1/4}(\bar{p}/\sqrt{\hat{p}}, -\sqrt{\hat{p}})^{\mathrm{T}}\,, \qquad \bar{\chi}_{\dot{\alpha}} = 2^{1/4}(\sqrt{\hat{p}}, p/\sqrt{\hat{p}})^{\mathrm{T}} \tag{2.7}$$

where $\chi^\alpha = \varepsilon^{\alpha\beta}\chi_\alpha$, $\bar{\chi}_{\dot{\alpha}} = \varepsilon_{\dot{\alpha}\dot{\beta}}\bar{\chi}^{\dot{\beta}}$ and $\varepsilon^{12} = \varepsilon^{\dot{1}\dot{2}} = -\varepsilon_{1,2} = -\varepsilon_{\dot{1}\dot{2}} = 1$.

The usual spinor bilinear products can be recast into our convention

$$\langle i\,j\rangle = \bar{\chi}_{\dot{\alpha}}(k_i)\bar{\chi}^{\dot{\alpha}}(k_j) = \sqrt{2}\frac{(i\,j)}{\sqrt{\hat{\imath}\,\hat{\jmath}}}\,, \tag{2.8}$$

$$[i\,j] = \chi^\alpha(k_i)\chi_\alpha(k_j) = -\sqrt{2}\frac{\{i\,j\}}{\sqrt{\hat{\imath}\,\hat{\jmath}}}\,. \tag{2.9}$$

### 2.3 SQCD Lagrangian and component fields

We will be working with the supersymmetric QCD Lagrangian with two chiral superfields:

$$
\begin{aligned}
L_{\mathrm{SQCD}} =& \int \mathrm{d}^3x \left[\Phi_1^\dagger e^{-2iV^*}\Phi_1 + \Phi_2^\dagger e^{-2iV}\Phi_2\right]\Big|_{\theta^4} - \frac{1}{2g^2}\mathrm{tr}\left[W^\alpha W_\alpha\Big|_{\theta^2} + h.c.\right] \\
&+m\left(\Phi_1\Phi_2\Big|_{\theta^2} + h.c.\right) \qquad (2.10)
\end{aligned}
$$

where $\Phi_2$ and $\Phi_1$ are the chiral superfields in the $N$ and $\bar{N}$ representation of color $\mathrm{SU}(N)$ group respectively and $W_\alpha$ is the field strength spinor superfield of the gauge field. $(\phi_1, \psi_1, F_1)$, $(\phi_2, \psi_2, F_2)$ are component fields of $\Phi_1$ and $\Phi_2$, and $(\Lambda_\alpha,\ \mathcal{F}_{\mu\nu}, D)$ are component fields of $W_\alpha$, $\mathcal{A}_\mu$ being the corresponding component gauge field in $V$:

$$
\begin{aligned}
\Phi(y) &= \phi(y) + \sqrt{2}\theta\psi(y) + \theta^2 F(y)\,, && (2.11)\\
W_\alpha(y) &= \Lambda_\alpha(y) + \theta^\beta\Big( - D(y)\varepsilon_{\beta\alpha} + i(\sigma^{\mu\nu})_\beta{}^\gamma\varepsilon_{\alpha\gamma}\mathcal{F}_{\mu\nu}(y)\Big) \\
&\quad +\theta^2(i\sigma^\mu_{\alpha\dot\alpha}\mathcal{D}_\mu\bar\Lambda^{\dot\alpha}(y))\,, && (2.12)
\end{aligned}
$$

where

$$
y^\mu = x^\mu - i\theta\sigma^\mu\bar\theta\,. \qquad (2.13)
$$

The gauge field strength is defined as

$$
\mathcal{F}_{\mu\nu} = [\mathcal{D}_\mu, \mathcal{D}_\nu], \quad \mathcal{D}_\mu = \partial_\mu + \mathcal{A}_\mu, \quad \mathcal{A}_\mu = -\frac{ig}{\sqrt{2}}A^a_\mu\tau^a\,, \quad T^a = -i\frac{\tau^a}{\sqrt{2}}, \qquad (2.14)
$$

where the normalization of color matrix $\tau^a$ is:

$$
[\tau^a, \tau^b] = i\sqrt{2}f^{abc}\tau^c, \quad \mathrm{tr}(\tau^a\tau^b) = \delta^{ab}. \qquad (2.15)
$$

$\mathcal{D}_\mu$ for gluino is defined as $\mathcal{D}_\mu\Lambda = \partial_\mu\Lambda + [\mathcal{A}_\mu, \Lambda]$.

Before gauge fixing, the SUSY transformations for component fields are:

$$
\begin{aligned}
\delta\mathcal{A}_\mu &= \eta\sigma_\mu\bar\Lambda + \Lambda\sigma_\mu\bar\eta && (2.16)\\
\delta\Lambda_\alpha &= D\eta_\alpha - i(\sigma^{\mu\nu})_\alpha{}^\beta\eta_\beta\mathcal{F}_{\mu\nu} \\
\delta D &= -i\eta\sigma^\mu\mathcal{D}_\mu\bar\Lambda - i\bar\eta\bar\sigma^\mu\mathcal{D}_\mu\Lambda \\
\delta\phi_{1,2} &= \sqrt{2}\eta^\alpha(\psi_{1,2})_\alpha\,, \\
\delta(\psi_{1,2})_\alpha &= \sqrt{2}(\eta_\alpha F_{1,2} - i[\sigma^\mu\bar\eta]_\alpha\mathcal{D}_\mu\phi_{1,2}) \\
\delta F_{1,2} &= -\sqrt{2}i\bar\eta\bar\sigma^\mu\mathcal{D}_\mu\psi_{1,2} - 2\bar\eta\bar\lambda\phi_{1,2}\,. && (2.17)
\end{aligned}
$$

After integrating out the auxilary fields $F$ and $D$ we find the SQCD Lagrangian in component fields:

$$
\begin{aligned}
L_{\mathrm{SQCD}} =& (\mathcal{D}_\mu\phi_1)^\dagger\mathcal{D}^\mu\phi_1 + (\mathcal{D}_\mu\phi_2)^\dagger\mathcal{D}^\mu\phi_2 - m^2(\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2) \\
&+i(\psi_1^\alpha \not{\mathcal{D}}_{\alpha\dot\alpha}\bar\psi_1^{\dot\alpha} + \bar\psi_2^{\dot\alpha}\not{\mathcal{D}}_{\dot\alpha\alpha}\bar\psi_2^\alpha) - m(\psi_1^\alpha\psi_{2\alpha} + \bar\psi_{1,\dot\alpha}\bar\psi_2^{\dot\alpha}) \\
&-\sqrt{2}i(\phi_1^{\mathrm{T}}\bar\Lambda\bar\psi_1 + \psi_1\Lambda\phi_1^*) + \sqrt{2}i(\bar\psi_2\bar\Lambda\phi_2 + \phi_2^\dagger\Lambda\psi_2)
\end{aligned}
$$

$$-\frac{2}{g^2}\mathrm{tr}\Big[-\frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}+\frac{i}{2}(\Lambda^{\alpha}\not{\mathcal{D}}_{\alpha\dot{\alpha}}\bar{\Lambda}^{\dot{\alpha}}+\bar{\Lambda}^{\dot{\alpha}}\not{\mathcal{D}}_{\dot{\alpha}\alpha}\Lambda^{\alpha})\Big]$$
$$+g^2(-\phi_1^{\mathrm{T}}T^a\phi_1^*+\phi_2^{\dagger}T^a\phi_2)^2\,. \tag{2.18}$$

We can combine the two Weyl spinors to be a Dirac spinor

$$\Psi=(\psi_{2,\alpha},\bar{\psi}_1^{\dot{\alpha}})^{\mathrm{T}} \tag{2.19}$$

and like in [29], we denote the components of the spinor as

$$\Psi=(\alpha^+,\beta^+,\beta^-,\alpha^-)^{\mathrm{T}}\quad\text{and}\quad\bar{\Psi}=(\bar{\beta}^+,\bar{\alpha}^+,\bar{\alpha}^-,\bar{\beta}^-)\,, \tag{2.20}$$

where the $\pm$ superscripts denote the physical helicity of the outgoing particles for massless theory and $\bar{\alpha}^{\pm}=(\alpha^{\mp})^*$. For the gluino, we denote the components of Weyl spinor $\Lambda$ and $\bar{\Lambda}$ as

$$\Lambda_\alpha=(\Lambda,T)\,,\quad\bar{\Lambda}^{\dot{\alpha}}=(\bar{T},-\bar{\Lambda})\,. \tag{2.21}$$

We also denote $\phi_1(\phi_1^*)$ as $\bar{\phi}^+(\bar{\phi}^-)$ and $\phi_2(\phi_2^*)$ as $\phi^+(\phi^-)$. In this way, we will see later that the superscript $\pm$ of $\alpha$, $\bar{\alpha}$ will be the same as their superpartners'. As an abuse of the nomenclature we will also call $\pm$ superscripts of scalars as plus(minus)-chirality.

Fields in momentum space are defined as the Fourier transformation:

$$f(x)=\int\frac{\mathrm{d}\hat{q}\,\mathrm{d}\tilde{q}\,\mathrm{d}\bar{q}}{(2\pi)^3}f(\vec{q})e^{i(\hat{q}x^{\bar{0}}+\tilde{q}x^z+\bar{q}x^{\bar{z}})} \tag{2.22}$$

and we use numbered subscripts to denote the momenta labelled with numbers of the fields:

$$f_1=f(\vec{p}_1)\,,\quad f_{\bar{1}}=f(-\vec{p}_1)\,. \tag{2.23}$$

We also use a short-hand notation for the momentum integral product

$$\int_{1\cdots n}=\prod_{k=1}^{n}\frac{1}{(2\pi)^3}\int d\hat{k}\;dk\;d\bar{k}. \tag{2.24}$$

## 3. Light-cone SQCD Lagrangian and light-cone SUSY transformations

We start with SQCD Lagrangian (2.18). As in [22–24, 29], we quantise the theory on the constant $x^0$ surface $\Sigma$ with a normal vector $\mu=(1,0,0,1)/\sqrt{2}$ in Minkowski co-ordinates and choose the light-cone gauge $\hat{\mathcal{A}}=0$. Then we find out that the dynamical fields are $\bar{\mathcal{A}}$, $\mathcal{A}$, $\bar{\alpha}^{\pm}$, $\alpha^{\pm}$, $\Lambda$, $\bar{\Lambda}$, $\bar{\phi}^{\pm}$, $\phi^{\pm}$, whilst $\check{\mathcal{A}}$, $T$, $\bar{T}$, $\beta^{\pm}$, $\bar{\beta}^{\pm}$ can be integrated out. After this, we can group the terms in the light-cone SQCD Lagrangian according to their chirality configuration and their field content:

$$\begin{aligned}L_{\mathrm{LCSQCD}}=\;&L_A^{+-}+L_\Lambda^{+-}+L_\phi^{+-}+L_\alpha^{+-}\\&+L_A^{++-}+L_{\Lambda A}^{++-}+L_{\alpha A}^{++-}+L_{\phi A}^{++-}+L_{\alpha\Lambda\phi}^{++-}\\&+L_A^{--+}+L_{\Lambda A}^{--+}+L_{\alpha A}^{--+}+L_{\phi A}^{--+}+L_{\alpha\Lambda\phi}^{--+}\\&+L_A^{--++}+L_\Lambda^{--++}+L_{\Lambda A}^{--++}+L_\alpha^{--++}+L_\phi^{--++}+L_{\alpha\phi}^{--++}\end{aligned}$$

$$
\begin{aligned}
&+L_{\alpha A}^{--++} + L_{\phi A}^{--++} + L_{\alpha\Lambda}^{--++} + L_{\phi\Lambda}^{--++} + L_{\alpha\phi\Lambda A}^{--++} \\
&+L_{m,\alpha}^{+-} + L_{m,\phi}^{+-} + L_{m,\alpha A}^{+-+} + L_{m,\phi\Lambda\alpha}^{+-+} + L_{m,\alpha A}^{-+-} + L_{m,\phi\Lambda\alpha}^{-+-} \,, \qquad (3.1)
\end{aligned}
$$

in which the superscripts differentiate their chirality configurations and the subscripts denote their field content. Subscript $m$ in the last line labels massive terms. $L_{m,\cdots}^{+-}$ terms are proportional to $m^2$, whilst $L_{m,\cdots}^{+-+}$ and $L_{m,\cdots}^{-+-}$ are proportional to $m$. The full expressions for each term are summarised in appendix A. The $L_{\alpha}^{+-}$, $L_{m,\alpha}^{+-}$ and $L_{\phi}^{+-}$, $L_{m,\phi}^{+-}$ terms result in massive propagators for $\alpha$ and $\phi$:

$$
\langle \alpha^{\pm} \bar{\alpha}^{\mp} \rangle = \frac{i\sqrt{2}\hat{p}}{p^2 - m^2} \,, \quad \langle \phi^- \phi^+ \rangle = \langle \bar{\phi}^- \bar{\phi}^+ \rangle = \frac{i}{p^2 - m^2} \,. \qquad (3.2)
$$

Since we have fixed the light-cone gauge, the full SUSY transformation will not preserve the gauge, but there is a subgroup of the full SUSY transfromation which leaves the light-cone gauge invariant. Also, since only the dynamical fields are left in the light-cone SQCD Lagrangian, the remaining supersymmetry can only involve these fields. If one restricts the SUSY transformation parameters $\eta_\alpha$ and $\bar{\eta}^{\dot{\alpha}}$ in (2.17) to be

$$
\eta_\alpha = (0, \eta) \,, \quad \bar{\eta}^{\dot{\alpha}} = (\bar{\eta}, 0) \,, \qquad (3.3)
$$

one finds that this subgroup of the full SUSY transformation does indeed preserve the gauge condition and the space of dynamical fields. To be specific, these transformations for dynamical fields are

$$
\begin{aligned}
\delta\bar{\phi}^+ &= -\sqrt{2}\eta\bar{\alpha}^+ \,, & \delta\bar{\phi}^- &= \sqrt{2}\bar{\eta}\alpha^- \,, & (3.4)\\
\delta\phi^+ &= \sqrt{2}\eta\alpha^+ \,, & \delta\phi^- &= -\sqrt{2}\bar{\eta}\bar{\alpha}^- \,, & (3.5)\\
\delta\bar{\alpha}^- &= 2i\eta\hat{\partial}\phi^- \,, & \delta\alpha^+ &= -2i\bar{\eta}\hat{\partial}\phi^+ \,, & (3.6)\\
\delta\alpha^- &= -2i\eta\hat{\partial}\bar{\phi}^- \,, & \delta\bar{\alpha}^+ &= 2i\bar{\eta}\hat{\partial}\bar{\phi}^+ \,, & (3.7)\\
\delta\Lambda &= 2i\eta\hat{\partial}\mathcal{A} \,, & \delta\bar{\Lambda} &= -2i\bar{\eta}\hat{\partial}\bar{\mathcal{A}} \,, & (3.8)\\
\delta\bar{\mathcal{A}} &= \sqrt{2}\eta\bar{\Lambda} \,, & \delta\mathcal{A} &= -\sqrt{2}\bar{\eta}\Lambda \,. & (3.9)
\end{aligned}
$$

The auxiliary fields $F$, $D$ and the nonlinear terms also automatically disappear in this transformation. We will call this SUSY transformation the light-cone SUSY transformation. Furthermore, we can group the dynamical fields into pairs:

$$
\{\alpha^+, \phi^+\} \,, \{\alpha^-, \bar{\phi}^-\} \,, \{\bar{\alpha}^-, \phi^-\} \,, \{\bar{\alpha}^+, \bar{\phi}^+\} \,, \{\Lambda, \mathcal{A}\}, \{\bar{\Lambda}, \bar{\mathcal{A}}\} \qquad (3.10)
$$

in which each pair of the dynamical fields generate an invariant sub-space under this light-cone SUSY transformation. As a result, the terms in the light-cone SQCD can also be grouped into SUSY invariant pieces:

$$
\begin{aligned}
&\{L_A^{+-} + L_\Lambda^{+-}\} \,, & &\{L_\phi^{+-} + L_\alpha^{+-}\} \,, \\
&\{L_A^{++-} + L_{\Lambda A}^{++-}\} \,, & &\{L_{\alpha A}^{++-} + L_{\phi A}^{++-} + L_{\alpha\Lambda\phi}^{++-}\} \,, \\
&\{L_A^{--+} + L_{\Lambda A}^{--+}\} \,, & &\{L_{\alpha A}^{--+} + L_{\phi A}^{--+} + L_{\alpha\Lambda\phi}^{--+}\} \,,
\end{aligned}
$$

$$\{L_A^{--++} + L_\Lambda^{--++} + L_{\Lambda A}^{--++}\}\,, \qquad \{L_\alpha^{--++} + L_\phi^{--++} + L_{\alpha\phi}^{--++}\}\,,$$
$$\{L_{\alpha A}^{--++} + L_{\phi A}^{--++} + L_{\alpha\Lambda}^{--++} + L_{\phi\Lambda}^{--++} + L_{\alpha\phi\Lambda A}^{--++}\}\,,$$
$$\{L_{m,\alpha}^{+-} + L_{m,\phi}^{+-}\}\,, \quad \{L_{m,\alpha A}^{+-+} + L_{m,\phi\Lambda\alpha}^{+-+}\}\,, \qquad \{L_{m,\alpha A}^{-+-} + L_{m,\phi\Lambda\alpha}^{-+-}\}\,. \tag{3.11}$$

These are not the smallest pieces that are invariant under these transformations. For example, one can also separate the $L_\phi^{+-} + L_\alpha^{+-}$ into two parts containing $\{\phi^\pm, \alpha^+, \bar{\alpha}^-\}$ and $\{\bar{\phi}^\pm, \alpha^-, \bar{\alpha}^+\}$ respectively which are separately invariant under the light-cone SUSY transformations. However, if one term in one curly bracket of (3.11) appears as a whole, to be closed under these transformations one must add the other terms in this curly bracket.

## 4. Canonical transformations for the MHV-SQCD

From previous work [29], we have already got the canonical transformations for $\alpha^\pm$, $\bar{\alpha}^\pm$, $\mathcal{A}$ and $\bar{\mathcal{A}}$ for the MHV Lagrangian of QCD. And now we also have the light-cone SUSY transformation (3.4)–(3.9). In this section, we will fix the canonical transformations for the SQCD using these two results.

The canonical field pairs before and after the canonical transformation for SQCD are (up to some irrelavent normalizations):

$$\{\mathcal{A}, \hat{\partial}\bar{\mathcal{A}}\} \to \{\mathcal{B}, \hat{\partial}\bar{\mathcal{B}}\}\,, \qquad \{\Lambda, \bar{\Lambda}\} \to \{\Pi, \bar{\Pi}\}\,,$$
$$\{\alpha^\pm, \bar{\alpha}^\mp\} \to \{\xi^\pm, \bar{\xi}^\mp\}\,,$$
$$\{\phi^-, \hat{\partial}\phi^+\} \to \{\varphi^-, \hat{\partial}\varphi^+\}\,, \qquad \{\bar{\phi}^-, \hat{\partial}\bar{\phi}^+\} \to \{\bar{\varphi}^-, \hat{\partial}\bar{\varphi}^+\}. \tag{4.1}$$

As in [22, 23, 29], the canonical transformation should transform the massless non-MHV terms to kinetic terms:

$$\begin{aligned} &L_A^{+-} + L_\Lambda^{+-} + L_\phi^{+-} + L_\alpha^{+-} + L_A^{++-} + L_{\Lambda A}^{++-} \\ &\qquad + L_{\alpha A}^{++-} + L_{\phi A}^{++-} + L_{\alpha\Lambda\phi}^{++-} = L_B^{+-} + L_\Pi^{+-} + L_\varphi^{+-} + L_\xi^{+-}\,. \end{aligned} \tag{4.2}$$

The canonical transformation can be represented as a power expansion of the old fields in terms of the new fields. Like equations (3.4)–(3.9), we expect the SUSY transformation for the new fields to be linear, so that it is satisfied at each order of the expansion. Since to the leading order of the transformation expansion, the old fields are the same as the new fields, we make the natural assumption that the new fields actually satisfy the same light-cone SUSY transformations as the corresponding old fields:

$$\delta\bar{\varphi}^+ = -\sqrt{2}\eta\bar{\xi}^+\,, \qquad \delta\bar{\varphi}^- = \sqrt{2}\bar{\eta}\xi^-\,, \tag{4.3}$$
$$\delta\varphi^+ = \sqrt{2}\eta\xi^+\,, \qquad \delta\varphi^- = -\sqrt{2}\bar{\eta}\bar{\xi}^-\,, \tag{4.4}$$
$$\delta\xi^- = -2i\eta\hat{\partial}\bar{\varphi}^-\,, \qquad \delta\bar{\xi}^+ = 2i\bar{\eta}\hat{\partial}\bar{\varphi}^+\,, \tag{4.5}$$
$$\delta\bar{\xi}^- = 2i\eta\hat{\partial}\varphi^-\,, \qquad \delta\xi^+ = -2i\bar{\eta}\hat{\partial}\varphi^+\,, \tag{4.6}$$
$$\delta\Pi = 2i\eta\hat{\partial}\mathcal{B}\,, \qquad \delta\bar{\Pi} = -2i\bar{\eta}\hat{\partial}\bar{\mathcal{B}}\,, \tag{4.7}$$
$$\delta\bar{\mathcal{B}} = \sqrt{2}\eta\bar{\Pi}\,, \qquad \delta\mathcal{B} = -\sqrt{2}\bar{\eta}\Pi\,. \tag{4.8}$$

As further support for this assumption, we note that this implies that both the left hand side and the right hand side of (4.2) are then closed under the light-cone SUSY transformation.

Recall that the equation defining the canonical transformation equation for MHV-QCD is only part of equation (4.2):

$$L_{A}^{+-} + L_{\alpha}^{+-} + L_{A}^{++-} + L_{\alpha A}^{++-} = L_{B}^{+-} + L_{\xi}^{+-} \,. \tag{4.9}$$

Since each term on the left hand side of this equation is in the first four curly brackets separately in (3.11) and also each term on the right hand side is in the first two curly brackets separately, the minimal supersymmetric extension of this equation just gives us back the whole equation (4.2). Therefore, if we can find a canonical transformation that transforms correctly under the SUSY transformation and is also consistent with what we already have in MHV-QCD, we will have solved the canonical transformation equation (4.2). To be specific, we can separate the transformation expansion for fields $\{\mathcal{A}, \bar{\mathcal{A}}, \alpha^{\pm}, \bar{\alpha}^{\pm}\}$ into QCD pieces which involve only QCD fields $\{\mathcal{B}, \bar{\mathcal{B}}, \xi^{\pm}, \bar{\xi}^{\pm}\}$, and new pieces which contain new supersymmetric fields $\{\varphi^{\pm}, \bar{\varphi}^{\pm}, \Pi, \bar{\Pi}\}$ as well as QCD fields:

$$X = X^{\mathrm{QCD}}[\mathcal{B}, \bar{\mathcal{B}}, \xi^{\pm}, \bar{\xi}^{\pm}] + X^{\mathrm{New}}[\mathcal{B}, \bar{\mathcal{B}}, \xi^{\pm}, \bar{\xi}^{\pm}, \varphi^{\pm}, \bar{\varphi}^{\pm}, \Pi, \bar{\Pi}] \,, \quad X \in \{\mathcal{A}, \bar{\mathcal{A}}, \alpha^{\pm}, \bar{\alpha}^{\pm}\}, \tag{4.10}$$

in which either each expansion term in $X^{\mathrm{New}}$ contains at least one field from $\{\varphi^{\pm}, \bar{\varphi}^{\pm}, \Pi, \bar{\Pi}\}$ or $X^{\mathrm{New}} = 0$. At the same time, one can see that the expansion of $\{\Lambda, \bar{\Lambda}, \phi^{\pm}, \bar{\phi}^{\pm}\}$ should not contain pieces which only have QCD fields:

$$\mathcal{X} = \mathcal{X}[\mathcal{B}, \bar{\mathcal{B}}, \xi^{\pm}, \bar{\xi}^{\pm}, \varphi^{\pm}, \bar{\varphi}^{\pm}, \Pi, \bar{\Pi}] \,, \quad \mathcal{X} \in \{\Lambda, \bar{\Lambda}, \phi^{\pm}, \bar{\phi}^{\pm}\}, \tag{4.11}$$

where each term in $\mathcal{X}$ contains at least one field from $\{\varphi^{\pm}, \bar{\varphi}^{\pm}, \Pi, \bar{\Pi}\}$ as in $X^{\mathrm{New}}$. This is because all the new superpartners carry $R$ charges, and $R$ charge should be conserved by the canonical transformation. In other words, $X^{\mathrm{New}}[\mathcal{B}, \bar{\mathcal{B}}, \xi^{\pm}, \bar{\xi}^{\pm}, \varphi^{\pm}, \bar{\varphi}^{\pm}, \Pi, \bar{\Pi}]$ and $\mathcal{X}[\mathcal{B}, \bar{\mathcal{B}}, \xi^{\pm}, \bar{\xi}^{\pm}, \varphi^{\pm}, \bar{\varphi}^{\pm}, \Pi, \bar{\Pi}]$ cannot contribute to the QCD pieces containing only QCD fields in the canonical transformation equation (4.2). Therefore $X^{\mathrm{QCD}}$ must separately satisfy (4.9), whilst the new pieces $X^{\mathrm{New}}$ along with $\mathcal{X}$ must satisfy the canonical transformation equation with the pure QCD terms eliminated:

$$\begin{aligned} L_{A}^{+-} + L_{\Lambda}^{+-} + L_{\phi}^{+-} + L_{\alpha}^{+-} + L_{A}^{++-} + L_{\Lambda A}^{++-} \\ + L_{\alpha A}^{++-} + L_{\phi A}^{++-} + L_{\alpha\Lambda\phi}^{++-} = L_{\varphi}^{+-} + L_{\Pi}^{+-} \,. \end{aligned} \tag{4.12}$$

Here only terms containing the new pieces $X^{\mathrm{New}}$ in $L_{A}^{+-}, L_{A}^{++-}, L_{\alpha}^{+-}, L_{\alpha A}^{++-}$ are retained. So we would expect $X^{\mathrm{QCD}}$ are just what we already have, which are listed in appendix B.

To obtain the MHV Lagrangian for massless pieces, we demand that all the old plus chirality fields depend only on new plus chirality fields and all the old minus chirality fields should depend linearly on the new minus chirality fields. As in [23, 29], first, we still demand that $\mathcal{A}$ is just a functional of $\mathcal{B}$, i.e. in momentum space:[1]

$$\mathcal{A}_q = \sum_{n=1}^{\infty} \int_{1\cdots n} \Upsilon_{q,\bar{1}\cdots\bar{n}} \mathcal{B}_1 \cdots \mathcal{B}_n \delta_{q\bar{1}\cdots\bar{n}} \,. \tag{4.13}$$

[1] Further details on definitions in the equations here and later, are included with the summaries in appendices B and C.

It is then easy to obtain the canonical transformation for $\Lambda$ by using the light-cone SUSY transformation (3.9) and (4.8):

$$\Lambda_q = \sum_{n=1}^{\infty} \int_{1\cdots n} \Upsilon_{q,\bar{1}\cdots\bar{n}} \sum_{l=1}^{n} \mathcal{B}_1 \cdots \Pi_l \cdots \mathcal{B}_n \delta_{q\bar{1}\cdots\bar{n}} \,. \tag{4.14}$$

Under SUSY, $\Lambda$ transforms back to $\mathcal{A}$ under the light-cone SUSY transformation. Therefore the right hand side of the above equation had better transform under SUSY back to the right hand side of (4.13). After collecting terms, that is what happens, thus forming a non-trivial consistency check on our reasoning. As an inverse, $\mathcal{B}$ is only a functional of $\mathcal{A}$, i.e. $\mathcal{B}[\mathcal{A}]$, and $\Pi$ a functional of $\mathcal{A}$ and $\Lambda$, i.e. $\Pi[\mathcal{A}, \Lambda]$.

Next, let us consider $\alpha^-$ and $\bar{\phi}^-$. The expansion of $\alpha^-$ should at least contain one piece containing terms of the form $\mathcal{B} \cdots \mathcal{B}\xi^-$. The supersymmetric transformation of $\alpha^-$ and $\xi^-$ involves only terms proportional to holomorphic $\eta$, but the supersymmetric transformation of $\mathcal{B}$ involves only anti-holomorphic $\bar{\eta}$. To satisfy the SUSY transformation for $\alpha^-$, the expansion must have another piece to cancel the $\bar{\eta}$ terms after the SUSY transformation. But for $\bar{\phi}^-$ there is no such requirement because the SUSY transformation of $\bar{\phi}^-$ involves only $\bar{\eta}$ like $\mathcal{B}$. So for a minimal extension of MHV-QCD, the expansion of $\bar{\phi}^-$ could contain only one piece:

$$\bar{\phi}^-_q = \sum_{n=1}^{\infty} \int_{1\cdots n} \Upsilon^-_{q,\bar{1}\cdots\bar{n}} \mathcal{B}_1 \cdots \mathcal{B}_{n-1} \bar{\varphi}^-_n \delta_{q\bar{1}\cdots\bar{n}} \,, \tag{4.15}$$

and by using the SUSY transformation from $\phi^-$ to $\alpha^-$, the expansion of $\alpha^-$ can be obtained:

$$\alpha^-_q = \xi^-_q + \sum_{n=2}^{\infty} \int_{1\cdots n} \Upsilon^-_{q,\bar{1}\cdots\bar{n}} \left( \mathcal{B}_1 \cdots \mathcal{B}_{n-1} \xi^-_n - \sum_{l=1}^{n-1} \mathcal{B}_1 \cdots \Pi_l \cdots \mathcal{B}_{n-1} \bar{\varphi}^-_n \right) \delta_{q\bar{1}\cdots\bar{n}} \,. \tag{4.16}$$

Under SUSY, $\alpha^-$ transforms back to $\bar{\phi}^-$. After collecting terms, one finds that the right hand side of the above equation transforms under SUSY back to the right hand side of (4.15). Again, this forms a non-trivial consistency check on our reasoning. As a result, the inverse $\bar{\varphi}^-$ is only a functional of $\bar{\phi}^-$ and $\mathcal{A}$, i.e. $\bar{\varphi}^-[\bar{\phi}^-, \mathcal{A}]$, and $\xi^-$ a functional of $\bar{\phi}^-$, $\Lambda$, $\alpha^-$, and $\mathcal{A}$, i.e. $\xi^-[\alpha^-, \bar{\phi}^-, \Lambda, \mathcal{A}]$. The discussion for $\bar{\alpha}^-$ and $\phi^-$ is the same. For $\bar{\alpha}^+$, $\bar{\phi}^+$ and $\alpha^+$, $\phi^+$, a similar discussion can be applied, but the roles of $\alpha$ and $\phi$ are exchanged, that is, the expansions for $\alpha^+$ and $\bar{\alpha}^+$ contain only one piece each and the expansions $\phi^+$ and $\bar{\phi}^+$ have two pieces each. The detailed transformations are summarized in appendix C. Here we only summarize the field dependence:

$$\begin{array}{llll} \alpha^+[\xi^+, \mathcal{B}] \,, & \bar{\alpha}^+[\bar{\xi}^+, \mathcal{B}] \,, & \alpha^-[\xi^-, \bar{\varphi}^-, \Pi, \mathcal{B}] \,, & \bar{\alpha}^-[\bar{\xi}^-, \varphi^-, \Pi, \mathcal{B}] \,, \\ \phi^-[\varphi^-, \mathcal{B}] \,, & \bar{\phi}^-[\bar{\varphi}^-, \mathcal{B}] \,, & \phi^+[\varphi^+, \xi^+, \Pi, \mathcal{B}] \,, & \bar{\phi}^+[\bar{\varphi}^+, \bar{\xi}^+, \Pi, \mathcal{B}] \,, \end{array} \tag{4.17}$$

and the inverse field dependence:

$$\begin{array}{llll} \xi^+[\alpha^+, \mathcal{A}] \,, & \bar{\xi}^+[\bar{\alpha}^+, \mathcal{A}] \,, & \xi^-[\alpha^-, \bar{\phi}^-, \Lambda, \mathcal{A}] \,, & \bar{\xi}^-[\bar{\alpha}^-, \phi^-, \Lambda, \mathcal{A}] \,, \\ \varphi^-[\phi^-, \mathcal{A}] \,, & \bar{\varphi}^-[\bar{\phi}^-, \mathcal{A}] \,, & \varphi^+[\phi^+, \alpha^+, \Lambda, \mathcal{A}] \,, & \bar{\varphi}^+[\bar{\phi}^+, \bar{\alpha}^+, \Lambda, \mathcal{A}] \,. \end{array} \tag{4.18}$$



JHEP12(2008)028

The transformations for $\bar{\Lambda}$ and $\bar{\mathcal{A}}$ are the most complicated ones. From the transformation for MHV-QCD we can see that, $\bar{\mathcal{A}}$ has at least two pieces: one piece involves only gauge fields $\mathcal{B}$ and $\bar{\mathcal{B}}$, and the other involves fermions and their $1/N_C$ terms. We look at the term involving $\mathcal{B}$ and $\bar{\mathcal{B}}$ i.e. $\bar{\mathcal{A}}^B$ first. From a similar holomorphic analysis of the SUSY transformation, one finds that the minimal extension is to make the corresponding gauge piece $\bar{\Lambda}^{\Pi B}$ of $\bar{\Lambda}$ contain a single piece involving $\mathcal{B}$ and $\bar{\Pi}$, and $\bar{\mathcal{A}}$ contain an additional piece $\mathcal{A}^{B\Pi}$ involving $\mathcal{B}$, $\Pi$, $\bar{\Pi}$, in the following sense:

$$\bar{\Lambda}^{B\Pi}_q = \sum_{n=1}^{\infty} \int_{1\cdots n} \sum_{l=1}^{n} \Xi^{l}_{q,\bar{1}\cdots\bar{n}} \mathcal{B}_1 \cdots \bar{\Pi}_l \cdots \mathcal{B}_n \delta_{q\bar{1}\cdots\bar{n}} \,, \tag{4.19}$$

$$\bar{\mathcal{A}}^{B\Pi}_q = -\frac{1}{\sqrt{2}\hat{q}} \sum_{n=2}^{\infty} \int_{1\cdots n} \sum_{s=1}^{n} \left[ \Xi^{s}_{q,\bar{1}\cdots\bar{n}} \sum_{l=1, l\neq s}^{n} (-1)^{\delta_{ls}} \mathcal{B}_1 \cdots \Pi_l \cdots \bar{\Pi}_s \cdots \mathcal{B}_n \right] \delta_{q\bar{1}\cdots\bar{n}} \,. \tag{4.20}$$

Next, let us look at the piece involving $\mathcal{B}\cdots\xi^{+}\bar{\xi}^{-}\cdots\mathcal{B}$ in the expansion of $\bar{\mathcal{A}}$. There must be a corresponding piece in $\bar{\Lambda}$ which should be transformed into this piece under the SUSY transformation. The corresponding piece in $\bar{\Lambda}$ could contain terms proportional to $\mathcal{B}\cdots\Pi\cdots\xi^{+}\bar{\xi}^{-}\cdots\mathcal{B}$, $\mathcal{B}\cdots\varphi^{+}\bar{\xi}^{-}\cdots\mathcal{B}$ and $\mathcal{B}\cdots\xi^{+}\varphi^{-}\cdots\mathcal{B}$. From (4.18) and the requirement that this be a canonical transformation, we have:

$$\frac{\delta\bar{\Lambda}}{\delta\hat{\partial}\varphi^{+}} = -\frac{\delta\varphi^{-}}{\delta\Lambda} = 0 \,, \quad \frac{\delta\bar{\Lambda}}{\delta\bar{\xi}^{-}} = \frac{\delta\xi^{+}}{\delta\Lambda} = 0 \,. \tag{4.21}$$

Therefore this piece cannot depend on $\varphi^{+}$ and $\bar{\xi}^{-}$, and the only terms left are proportional to $\mathcal{B}\cdots\xi^{+}\varphi^{-}\cdots\mathcal{B}$. This is also consistent with the SUSY transformation since the SUSY transformations of $\mathcal{B}$, $\varphi^{-}$ and $\xi^{+}$ involve only $\bar{\eta}$ which is the same for $\bar{\Lambda}$. By the same reasoning, one can obtain the corresponding pieces in $\bar{\Lambda}$ for the other pieces in $\bar{\mathcal{A}}$. In this way we demonstrate that $\bar{\Lambda}$ must be given by equations (C.9)–(C.11). Now, using SUSY transformation on $\bar{\Lambda}$ one determines the canonical transformation for $\bar{\mathcal{A}}$ as given in the equations (C.12)–(C.16). Finally, by a straightforward computation, one confirms that the expansion for $\bar{\mathcal{A}}$ transforms back to the expansion for $\bar{\Lambda}$ under the SUSY transformations.

We have now determined the canonical transformations for all of the fields in terms of coefficients which we have tacitly assumed are the same as the ones in the transformation to MHV-QCD. This assumption is correct, since the QCD pieces of $\{\mathcal{A}, \bar{\mathcal{A}}, \alpha^{\pm}, \bar{\alpha}^{\pm}\}$ must be just the same as the transformation for MHV-QCD. It follows that the canonical transformations for new fields does not introduce new unknown coefficients, as one might expect from supersymmetry.

## 5. The massive CSW vertices for SQCD

With the canonical transformations at hand, we are ready to look at the new CSW vertices for SQCD. As proved in [22, 29], the massless part of the CSW vertices are the same as the MHV amplitude continued off shell up to some external polarizations. These MHV vertices can be constructed from the MHV amplitudes obtained using normal massless SUSY Ward



JHEP12(2008)028

Identities (SWI), so we will not discuss them here. The new vertices are the massive CSW vertices from mass terms in the light-cone SQCD.

The new massive CSW vertices for SQCD could be calculated by substituting the canonical transformations into the light-cone SQCD Lagrangian. But we can try to fix them from the light-cone SUSY transformation. First let us look at

$$L^{+-}_{m,\alpha\phi} = L^{+-}_{m,\alpha} + L^{+-}_{m,\phi}. \tag{5.1}$$

Upon substitution of the canonical transformation, we see that this piece should be composed of six kinds of field configurations:

$$\begin{aligned}
L^{+-}_{m,\alpha\phi} = \frac{m^2}{\sqrt{2}}\sum_{n=2}^{\infty}\int_{1\cdots n}\Big[&V^{+-}_{m,\xi}(\bar{1}\cdots\bar{n})\bar{\xi}^+_1\mathcal{B}_2\cdots\mathcal{B}_{n-1}\xi^-_n + V^{-+}_{m,\xi}(\bar{1}\cdots\bar{n})\,\bar{\xi}^-_1\mathcal{B}_2\cdots\mathcal{B}_{n-1}\xi^+_n \\
&+V^{+-}_{m,\varphi}(\bar{1}\cdots\bar{n})\bar{\varphi}^+_1\mathcal{B}_2\cdots\mathcal{B}_{n-1}\bar{\varphi}^-_n + V^{-+}_{m,\varphi}(\bar{1}\cdots\bar{n})\varphi^-_1\mathcal{B}_2\cdots\mathcal{B}_{n-1}\varphi^+_n\Big]\delta_{1\cdots n} \\
+\frac{m^2}{\sqrt{2}}\sum_{n=3}^{\infty}\int_{1\cdots n}\sum_{l=2}^{n-1}\Big[&V^{l,+-}_{m,\xi\Pi\varphi}(\bar{1}\cdots\bar{n})\,\bar{\xi}^+_1\mathcal{B}_2\cdots\Pi_l\cdots\mathcal{B}_{n-1}\bar{\varphi}^-_n \\
&+V^{l,-+}_{m,\varphi\Pi\xi}(\bar{1}\cdots\bar{n})\,\varphi^-_1\mathcal{B}_2\cdots\Pi_l\cdots\mathcal{B}_{n-1}\xi^+_n\Big]\delta_{1\cdots n}\,.
\end{aligned} \tag{5.2}$$

Note that there are no $\bar{\varphi}^+\mathcal{B}\cdots\Pi\cdots\mathcal{B}\xi^-$ and $\bar{\xi}^-\mathcal{B}\cdots\Pi\cdots\mathcal{B}\varphi^+$ terms. This is a consequence of the simple field dependence of $\alpha^+$, $\bar{\alpha}^+$, $\phi^-$, and $\bar{\phi}^-$ in (4.17). It also conforms to a rule that one can extract from the light-cone SQCD Lagrangian in appendix A, namely that in the terms involving one scalar and one gluino, the chiralities of the scalar and gluino are always opposite. The first, third and fifth pieces should be closed under SUSY transformations. Using the SUSY transformations (4.3)–(4.8) one then easily finds that these three vertices must be related as:

$$V^{+-}_{m,\varphi} = -\sqrt{2}\,\hat{1}V^{+-}_{m,\xi}\,, \quad V^{l,+-}_{m,\xi\Pi\varphi} = V^{+-}_{m,\xi}\,. \tag{5.3}$$

Since only the QCD pieces in $\alpha^{\pm}$ and $\bar{\alpha}^{\pm}$ can contribute to terms proportional to $\bar{\xi}^+_1\mathcal{B}_2\cdots\mathcal{B}_{n-1}\xi^-_n$ and $\bar{\xi}^-_1\mathcal{B}_2\cdots\mathcal{B}_{n-1}\xi^+_n$, $V^{+-}_{m,\xi}$ and $V^{-+}_{m,\xi}$ must be equal to the corresponding coefficients for MHV-QCD. By using the results of $V^{+-}_{m,\xi}$ for MHV-QCD listed in the appendix D, the above relations immediately give $V^{+-}_{m,\varphi}$ and $V^{+-}_{m,\xi\Pi\varphi}$. Similarly, we can also obtain expressions for the other three vertices:

$$V^{-+}_{m,\varphi} = \sqrt{2}\,\hat{1}V^{-+}_{m,\xi}\,, \quad V^{l,-+}_{m,\varphi\Pi\xi} = V^{-+}_{m,\xi}\,. \tag{5.4}$$

If one calculates the amplitudes $A(1^{\pm}_{\mathrm{q}}2^+\cdots(n-1)^+n^{\mp}_{\bar{\mathrm{q}}})$ and $A(1^{\pm}_{\varphi}2^+\cdots(n-1)^+n^{\mp}_{\bar{\varphi}})$ by choosing the reference momenta of fermions as in [31], the amplitudes are proportional to $V^{\pm\mp}_{m,\xi}$ and $V^{\pm\mp}_{m,\varphi}$. Using relation (5.3) and (5.4), one can recover the relations between these amplitudes obtained from massive SWI in [31]. Notice that $V^{l,-+}_{m,\varphi\Pi\xi}$ and $V^{l,+-}_{m,\xi\Pi\varphi}$ are independent of $l$. This is a consequence of the supersymmetry. Taking $V^{l,+-}_{m,\xi\Pi\varphi}$ as an example, since after SUSY transformation, there is a term proportional to $\bar{\xi}^+_1\mathcal{B}_2\cdots\Pi_s\cdots\Pi_l\cdots\mathcal{B}\bar{\varphi}^-_n$

which comes from $V^{l,+-}_{m,\xi\Pi\varphi}\bar{\xi}^+_1\mathcal{B}_2\cdots\Pi_l\cdots\mathcal{B}\bar{\varphi}^-_n$ and $V^{s,+-}_{m,\xi\Pi\varphi}\bar{\xi}^+_1\mathcal{B}_2\cdots\Pi_s\cdots\mathcal{B}\bar{\varphi}^-_n$, whose coefficient must vanish, we have:

$$V^{l,+-}_{m,\xi\Pi\varphi}-V^{s,+-}_{m,\xi\Pi\varphi}=0\,,\quad \text{for } s\neq l\,, \tag{5.5}$$

in other words $V^{l,+-}_{m,\xi\Pi\varphi}$ does not depend on $l$.

Next, let us look at

$$L^{+-+}_m=L^{+-+}_{m,\alpha A}+L^{+-+}_{m,\phi A}+L^{+-+}_{m,\phi\Lambda\alpha}\,. \tag{5.6}$$

We can separate it into three parts which are invariant under the supersymmetry:

$$\begin{aligned}
L^{+-+,(1)}_m &= im\Bigg\{\sum_{n=3}^{\infty}\int_{1\cdots n}\sum_{s=2}^{n-1}\Big(V^{s,+-+}_{m,\xi}(\bar{1}\cdots\bar{n})\,\bar{\xi}^+_1\mathcal{B}_2\cdots\bar{\mathcal{B}}_s\cdots\mathcal{B}_{n-1}\xi^+_n\\
&\qquad +V^{s,+-+}_{m,\varphi\Pi\xi}(\bar{1}\cdots\bar{n})\bar{\varphi}^+_1\mathcal{B}_2\cdots\bar{\Pi}_s\cdots\mathcal{B}_{n-1}\xi^+_n\\
&\qquad +V^{s,+-+}_{m,\xi\Pi\varphi}(\bar{1}\cdots\bar{n})\,\bar{\xi}^+_1\mathcal{B}_2\cdots\bar{\Pi}_s\cdots\mathcal{B}_{n-1}\varphi^+_n\Big)\delta_{1\cdots n}\\
&\quad +\frac{1}{\sqrt{2}}\sum_{n=4}^{\infty}\int_{1\cdots n}\sum_{l=2}^{n-1}\sum_{s=2,s\neq l}^{n-1}(-1)^{\delta_{ls}}\Big(V^{ls,+-+}_{\xi\Pi\bar{\Pi}\xi}(\bar{1}\cdots\bar{n})\\
&\quad \times\bar{\xi}^+_1\mathcal{B}_2\cdots\Pi_l\cdots\bar{\Pi}_s\cdots\mathcal{B}_{n-1}\xi^+_n\Big)\delta_{1\cdots n}\Bigg\}\,,
\end{aligned} \tag{5.7}$$

$$\begin{aligned}
L^{+-+,(2)}_m &= i\frac{mg^2}{2\sqrt{2}}\Bigg\{\sum_{n=4}^{\infty}\int_{1\cdots n}\sum_{s=2}^{n-2}\Big(V^{s,++-+}_{m,\xi}(\bar{1}\cdots\bar{n})\bar{\xi}^+_1\mathcal{B}_2\cdots\xi^+_s\bar{\xi}^-_{s+1}\cdots\mathcal{B}_{n-1}\xi^+_n\\
&\qquad +V^{s,++-+}_{m,\xi\varphi\varphi\xi}(\bar{1}\cdots\bar{n})\,\bar{\xi}^+_1\mathcal{B}_2\cdots\varphi^+_s\varphi^-_{s+1}\cdots\mathcal{B}_{n-1}\xi^+_n\\
&\qquad +V^{s,++-+}_{m,\varphi\xi\varphi\xi}(\bar{1}\cdots\bar{n})\,\bar{\varphi}^+_1\mathcal{B}_2\cdots\xi^+_s\varphi^-_{s+1}\cdots\mathcal{B}_{n-1}\xi^+_n\\
&\qquad +V^{s,++-+}_{m,\xi\xi\varphi\varphi}(\bar{1}\cdots\bar{n})\bar{\xi}^+_1\mathcal{B}_2\cdots\xi^+_s\varphi^-_{s+1}\cdots\mathcal{B}_{n-1}\varphi^+_n\Big)\delta_{1\cdots n}\\
&\quad +\sum_{n=5}^{\infty}\int_{1\cdots n}\sum_{s=2}^{n-2}\sum_{l=2,l\neq s,s+1}^{n-1}(-1)^{\delta_{ls}}\Big(V^{ls,++-+}_{m,\xi\Pi\varphi\varphi\xi}(\bar{1}\cdots\bar{n})\\
&\quad \times\bar{\xi}^+_1\mathcal{B}_2\cdots\Pi_l\cdots\xi^+_s\varphi^-_{s+1}\cdots\mathcal{B}_{n-1}\xi^+_n\Big)\delta_{1\cdots n}\Bigg\}\,,
\end{aligned} \tag{5.8}$$

$$\begin{aligned}
L^{+-+,(3)}_m &= i\frac{mg^2}{2\sqrt{2}}\Bigg\{\sum_{n=4}^{\infty}\int_{1\cdots n}\sum_{s=2}^{n-2}\Big(V^{s,+-++}_{m,\xi}(\bar{1}\cdots\bar{n})\bar{\xi}^+_1\mathcal{B}_2\cdots\xi^-_s\bar{\xi}^+_{s+1}\cdots\mathcal{B}_{n-1}\xi^+_n\\
&\qquad +V^{s,+-++}_{m,\xi\varphi\varphi\xi}(\bar{1}\cdots\bar{n})\,\bar{\xi}^+_1\mathcal{B}_2\cdots\bar{\varphi}^-_s\bar{\varphi}^+_{s+1}\cdots\mathcal{B}_{n-1}\xi^+_n\\
&\qquad +V^{s,+-++}_{m,\varphi\varphi\xi\xi}(\bar{1}\cdots\bar{n})\bar{\varphi}^+_1\mathcal{B}_2\cdots\bar{\varphi}^-_s\bar{\xi}^+_{s+1}\cdots\mathcal{B}_{n-1}\xi^+_n\\
&\qquad +V^{s,+-++}_{m,\xi\varphi\xi\varphi}(\bar{1}\cdots\bar{n})\bar{\xi}^+_1\mathcal{B}_2\cdots\bar{\varphi}^-_s\bar{\xi}^+_{s+1}\cdots\mathcal{B}_{n-1}\varphi^+_n\Big)\delta_{1\cdots n}\\
&\quad +\sum_{n=5}^{\infty}\int_{1\cdots n}\sum_{s=2}^{n-2}\sum_{l=2,l\neq s,s+1}^{n-1}(-1)^{\delta_{ls}}\Big(V^{ls,+-++}_{m,\xi\Pi\varphi\xi\xi}(\bar{1}\cdots\bar{n})\\
&\quad \times\bar{\xi}^+_1\mathcal{B}_2\cdots\Pi_l\cdots\bar{\varphi}^-_s\bar{\xi}^+_{s+1}\cdots\mathcal{B}_{n-1}\xi^+_n\Big)\delta_{1\cdots n}\Bigg\}\,,
\end{aligned} \tag{5.9}$$

where $\delta_{ls}$ is defined in (C.17). Just by using the SUSY transformation as before, we arrive at the following relations between these vertices:

$$
\begin{aligned}
V^{s,+-+}_{m,\varphi\Pi\xi} &= \frac{\hat{1}}{\hat{s}} V^{s,+-+}_{m,\xi}\,, & V^{s,+-+}_{m,\xi\Pi\varphi} &= -\frac{\hat{n}}{\hat{s}} V^{s,+-+}_{m,\xi}\,, & V^{ls,+-+}_{m,\xi\Pi\bar{\Pi}\xi} &= -\frac{1}{\hat{s}} V^{s,+-+}_{m,\xi} \\
V^{s,++-+}_{m,\varphi\xi\varphi\xi} &= \sqrt{2}\hat{1} V^{s,++-+}_{m,\xi}\,, & V^{s,++-+}_{m,\xi\xi\varphi\varphi} &= -\sqrt{2}\hat{n} V^{s,++-+}_{m,\xi}\,, & & \\
V^{s,++-+}_{m,\xi\varphi\varphi\xi} &= \sqrt{2}\hat{s} V^{s,++-+}_{m,\xi}\,, & V^{ls,++-+}_{m,\xi\Pi\varphi\varphi\xi} &= -V^{s,++-+}_{m,\xi}\,, & & \\
V^{s,+-++}_{m,\xi\varphi\xi\varphi} &= -\sqrt{2}\hat{n} V^{s,+-++}_{m,\xi}\,, & V^{s,+-++}_{m,\varphi\varphi\xi\xi} &= \sqrt{2}\hat{1} V^{s,+-++}_{m,\xi}\,, & & \\
V^{s,+-++}_{m,\xi\varphi\varphi\xi} &= -\sqrt{2}\widehat{s+1} V^{s,+-++}_{m,\xi}\,, & V^{ls,+-++}_{m,\xi\Pi\varphi\varphi\xi} &= -V^{s,+-++}_{m,\xi}\,. & &
\end{aligned}
\tag{5.10}
$$

Since all the vertices on the right hand side of these equations have already been obtained from MHV-QCD, the new vertices on the left hand side immediately follow from these relations. Notice that $V^{ls,+-+}_{m,\xi\Pi\bar{\Pi}\xi}$, $V^{ls,++-+}_{m,\xi\Pi\varphi\varphi\xi}$ and $V^{ls,+-++}_{m,\xi\Pi\varphi\varphi\xi}$ also do not depend on $l$, so the dependence of $l$ in corresponding terms appears only in $(-1)^{\delta_{ls}}$, i.e. when the positions of plus-helicity gluinos change, the vertices at most change sign, in accordance with Fermi statistics. Just like in the previous case, this can be understood as a result of the supersymmetry: for example, the coefficient of the term $\bar{\xi}^+ \cdots \Pi_{l_1} \cdots \Pi_l \cdots \xi^+ \varphi^- \cdots \xi^+$, generated by the SUSY transformation, must vanish. This arises from transforming the $V^{ls,++-+}_{m,\xi\Pi\varphi\varphi\xi}$, $V^{l_1 s,++-+}_{m,\xi\Pi\varphi\varphi\xi}$ terms, from which we conclude

$$
V^{ls,++-+}_{m,\xi\Pi\varphi\varphi\xi} - V^{l_1 s,++-+}_{m,\xi\Pi\varphi\varphi\xi} = 0\,, \quad \text{for } l_1 \neq l\,,
\tag{5.11}
$$

which just means that $V^{ls,++-+}_{m,\xi\Pi\varphi\varphi\xi}$ is independent of $l$.

Now let us look at the last piece

$$
L^{-+-}_{m} = L^{-+-}_{m,\alpha A} + L^{-+-}_{m,\phi\Lambda\alpha}\,.
\tag{5.12}
$$

There are four kinds of vertices with different field configurations in the Lagrangian after applying the canonical transformations:

$$
\begin{aligned}
L^{-+-}_{m} = im \Bigg\{ & \sum_{n=3}^{\infty} \int_{1\cdots n} \Bigg[ V^{-+-}_{m,\xi}(\bar{1}\cdots\bar{n}) \bar{\xi}^-_1 \mathcal{B}_2 \cdots \mathcal{B}_{n-1} \xi^-_n \\
& \qquad + \sum_{l=2}^{n-1} V^{l,-+-}_{m,\varphi\Pi\xi}(\bar{1}\cdots\bar{n}) \varphi^-_1 \mathcal{B}_2 \cdots \Pi_l \cdots \mathcal{B}_{n-1} \xi^-_n \\
& \qquad + \sum_{l=2}^{n-1} V^{l,-+-}_{m,\xi\Pi\varphi}(\bar{1}\cdots\bar{n}) \bar{\xi}^-_1 \mathcal{B}_2 \cdots \Pi_l \cdots \mathcal{B}_{n-1} \bar{\varphi}^-_n \Bigg] \delta_{1\cdots n} \\
& + \sum_{n=4}^{\infty} \int_{1\cdots n} \sum_{l_1=2}^{n-2} \sum_{l_2=l_1+1}^{n-1} V^{l_1 l_2,-+-}_{m,\varphi\Pi\Pi\varphi}(\bar{1}\cdots\bar{n}) \\
& \times \varphi^-_1 \mathcal{B}_2 \cdots \Pi_{l_1} \cdots \Pi_{l_2} \cdots \mathcal{B}_{n-1} \bar{\varphi}^-_n \; \delta_{1\cdots n} \Bigg\}\,.
\end{aligned}
\tag{5.13}
$$

Unfortunately, the light-cone SUSY transformation can only provide some relations among these vertices:

$$\hat{1}\, V^{-+-}_{m,\xi} + \sum_{l=2}^{n-1} \hat{l}\, V^{l,-+-}_{m,\varphi\Pi\xi} = 0\,, \quad \hat{n}\, V^{-+-}_{m,\xi} - \sum_{l=2}^{n-1} \hat{l}\, V^{l,-+-}_{m,\xi\Pi\varphi} = 0\,, \tag{5.14}$$

$$V^{-+-}_{m,\xi} - V^{l,-+-}_{m,\varphi\Pi\xi} + V^{l,-+-}_{m,\xi\Pi\varphi} = 0\,, \tag{5.15}$$

$$\hat{n}\, V^{l,-+-}_{m,\varphi\Pi\xi} + \hat{1}\, V^{l,-+-}_{m,\xi\Pi\varphi} + \sum_{l_1=2}^{l-1} \hat{l}_1 V^{l_1 l,-+-}_{m,\varphi\Pi\Pi\varphi} - \sum_{l_1=l+1}^{n-1} \hat{l}_1 V^{l l_1,-+-}_{m,\varphi\Pi\Pi\varphi} = 0\,, \tag{5.16}$$

$$V^{l_2,-+-}_{m,\varphi\Pi\xi} - V^{l_1,-+-}_{m,\varphi\Pi\xi} - V^{l_1 l_2,-+-}_{m,\varphi\Pi\Pi\varphi} = 0\,, \tag{5.17}$$

$$V^{l_2,-+-}_{m,\xi\Pi\varphi} - V^{l_1,-+-}_{m,\xi\Pi\varphi} - V^{l_1 l_2,-+-}_{m,\varphi\Pi\Pi\varphi} = 0\,, \tag{5.18}$$

$$V^{l_1 l_2,-+-}_{m,\varphi\Pi\Pi\varphi} - V^{s l_2,-+-}_{m,\varphi\Pi\Pi\varphi} + V^{s l_1,-+-}_{m,\varphi\Pi\Pi\varphi} = 0\,. \tag{5.19}$$

From these relations, we cannot fix these vertices uniquely, but the above relations may simplify their determination. Indeed, we only need to calculate two of these vertices to obtain the others. Since we already have $V^{-+-}_{m,\xi}$ for MHV-QCD, actually we only need to calculate one vertex. The results are:

$$V^{l,-+-}_{m,\varphi\Pi\xi} = \frac{\hat{1}\,(l\, n)}{\hat{l}\,(1\, n)} V^{-+-}_{m,\xi}\,, \tag{5.20}$$

$$V^{l,-+-}_{m,\xi\Pi\varphi} = -\frac{\hat{n}\,(1\, l)}{\hat{l}\,(1\, n)} V^{-+-}_{m,\xi}\,, \tag{5.21}$$

$$V^{l_1 l_2,-+-}_{m,\varphi\Pi\Pi\varphi} = -\frac{\hat{1}\hat{n}\,(l_1\, l_2)}{\hat{l}_1\hat{l}_2\,(1\, n)} V^{-+-}_{m,\xi}\,. \tag{5.22}$$

It is easy to check that these equations satisfy (5.14)–(5.19). These relations in fact can also be understood using the massive SWI at the amplitude level [31].

## 6. Conclusion and discussion

In this paper, we have seen that the whole canonical transformation for the MHV-SQCD Lagrangian can be obtained simply by applying a kind of light-cone supersymmetry transformation on the canonical transformation for MHV-QCD Lagrangian. Unlike SWI, this SUSY transformation relates the dynamical fields of light-cone SQCD directly at the Lagrangian level, not the annihilation operators of outgoing states at the amplitude level. As a result, it is more widely applicable, as we saw for example by using it to uniquely determine the canonical transformation for MHV-SQCD. Some relations among massive CSW vertices can be understood using this supersymmetric transformation. Using these relations and the canonical transformation, all the massive CSW vertices are obtained. But since this SUSY transformation is a subgroup of the whole supersymmetry transformation, it is not as flexible as SWI in changing the transformation parameters. We see this in the massive vertex relations (5.20)–(5.22), which can be obtained from SWI at the amplitude level, but can not all be obtained just by using this SUSY transformation.



JHEP12(2008)028

Though in this paper, the canonical transformation is derived for MHV-SQCD with one flavour, clearly its application is more general. More flavours can be added without difficulty. Since the pieces having different field content in the canonical transformation equation (4.2) cancel separately from the left and right hand side of the equation, parts of the transformation can be directly used in theories which can be embedded in SQCD. A typical example is MHV-QCD. After turning off $\Lambda$ and $\phi_{1,2}$ by setting them to zero, we can obtain the canonical transformation for MHV-QCD and the corresponding massive CSW vertices are not changed. For theories involving only a gauge field and scalars, we can set $\Lambda$ and quark fields to be zero in the canonical transformation, and the corresponding parts of the massive CSW vertices can be used directly in this theory as in [27, 28]. This provides a direct explanation of the similarity between the massive CSW scalar vertices in [27, 28] and the massive CSW fermion vertices in [29]. Moreover, changing the masses for scalars and fermions does not modify the canonical transformation but the CSW vertices could be modified. It is also possible to extend it to a supersymmetric theory incorporating standard model such as the MSSM.

These CSW vertices of course can be used in simplifying calculations of SQCD amplitudes involving massive quarks and scalars. An interesting observation is that for terms involving a plus-helicity gluino and containing only one minus-chirality particle, the massive vertices do not depend on the position of the plus-helicity gluino in the color matrix product except for a possible sign change according to fermion statistics. This is a consequence of supersymmetry. This property may be useful in the amplitude calculations in SQCD.


## Acknowledgments

The authors thank the STFC for financial support.


## A. Light-cone Lagrangian for SQCD

$$L_{A}^{+-} = \frac{4}{g^2}\,\mathrm{tr}\int_{\Sigma} d^3\mathbf{x}\;\bar{\mathcal{A}}\,(\check{\partial}\hat{\partial} - \partial\bar{\partial})\,\mathcal{A}\,, \tag{A.1}$$

$$L_{\Lambda}^{+-} = -i\frac{2\sqrt{2}}{g^2}\mathrm{tr}\int_{\Sigma} d^3\mathbf{x}\;(\bar{\Lambda}\check{\partial}\Lambda - \bar{\Lambda}\bar{\partial}\hat{\partial}^{-1}\partial\Lambda)\,, \tag{A.2}$$

$$L_{\phi}^{+-} = -2\int_{\Sigma} d^3\mathbf{x}\;(\bar{\phi}^{+}(\hat{\partial}\check{\partial} - \partial\bar{\partial})\bar{\phi}^{-} + \phi^{-}(\hat{\partial}\check{\partial} - \partial\bar{\partial})\phi^{+})\,, \tag{A.3}$$

$$L_{\alpha}^{+-} = i\sqrt{2}\int_{\Sigma} d^3\mathbf{x}\;(\bar{\alpha}^{+}\check{\partial}\alpha^{-} + \bar{\alpha}^{-}\check{\partial}\alpha^{+} - \bar{\alpha}^{+}\bar{\partial}\hat{\partial}^{-1}\partial\alpha^{-} - \bar{\alpha}^{-}\partial\hat{\partial}^{-1}\bar{\partial}\alpha^{+})\,. \tag{A.4}$$

$$L_{A}^{++-} = -\frac{4}{g^2}\,\mathrm{tr}\int_{\Sigma} d^3\mathbf{x}\;(\bar{\partial}\hat{\partial}^{-1}\mathcal{A})\,[\mathcal{A},\,\hat{\partial}\bar{\mathcal{A}}]\,, \tag{A.5}$$

$$L_{\Lambda A}^{++-} = -i\frac{2\sqrt{2}}{g^2}\,\mathrm{tr}\int_{\Sigma} d^3\mathbf{x}\;\Big(\mathcal{A}\{\Lambda, \bar{\partial}\hat{\partial}^{-1}\Lambda\} - (\bar{\partial}\hat{\partial}^{-1}\mathcal{A})\{\Lambda, \bar{\Lambda}\}\Big)\,, \tag{A.6}$$

$$\begin{aligned} L_{\alpha A}^{++-} = i\sqrt{2}\int_{\Sigma} d^3\mathbf{x}\;\Big(&\bar{\alpha}^{+}(\hat{\partial}^{-1}\bar{\partial}\mathcal{A})\alpha^{-} + \bar{\alpha}^{-}(\hat{\partial}^{-1}\bar{\partial}\mathcal{A})\alpha^{+} \\ &-\bar{\alpha}^{+}\bar{\partial}\hat{\partial}^{-1}(\mathcal{A}\alpha^{-}) - \bar{\alpha}^{-}\mathcal{A}\hat{\partial}^{-1}\bar{\partial}\alpha^{+}\Big)\,, \end{aligned} \tag{A.7}$$

JHEP12(2008)028

$$L^{++-}_{\phi A} = -2\int_\Sigma d^3\mathbf{x}\,\Big(\bar\phi^+(\hat\partial^{-1}\bar\partial\mathcal{A})\hat\partial\bar\phi^- - \partial\phi^-(\hat\partial^{-1}\bar\partial\mathcal{A})\phi^+ - \bar\phi^+\mathcal{A}\bar\partial\bar\phi^- + \bar\partial\phi^-\mathcal{A}\phi^+\Big)\,, \tag{A.8}$$

$$L^{++-}_{\alpha\Lambda\phi} = i\sqrt{2}\int_\Sigma d^3\mathbf{x}\,\Big(-\bar\alpha^+(\hat\partial^{-1}\bar\partial\Lambda)\bar\phi^- + \phi^-(\hat\partial^{-1}\bar\partial\Lambda)\alpha^+ + (\hat\partial^{-1}\bar\partial\bar\alpha^+)\Lambda\bar\phi^- - \phi^-\Lambda(\hat\partial^{-1}\bar\partial\alpha^+)\Big)\,. \tag{A.9}$$

$$L^{--+}_{A} = -\frac{4}{g^2}\,\mathrm{tr}\int_\Sigma d^3\mathbf{x}\,[\bar{\mathcal{A}},\,\partial\mathcal{A}]\,(\partial\hat\partial^{-1}\bar{\mathcal{A}})\,, \tag{A.10}$$

$$L^{--+}_{\Lambda A} = -i\frac{2\sqrt{2}}{g^2}\,\mathrm{tr}\int_\Sigma d^3\mathbf{x}\,\Big(\bar{\mathcal{A}}\,\{\Lambda,\partial\hat\partial^{-1}\bar\Lambda\} - (\partial\hat\partial^{-1}\bar{\mathcal{A}})\{\bar\Lambda,\Lambda\}\Big)\,, \tag{A.11}$$

$$L^{--+}_{\alpha A} = i\sqrt{2}\int_\Sigma d^3\mathbf{x}\,\Big(\bar\alpha^+(\hat\partial^{-1}\partial\bar{\mathcal{A}})\alpha^- + \bar\alpha^-(\hat\partial^{-1}\partial\bar{\mathcal{A}})\alpha^+ - \bar\alpha^-\partial\hat\partial^{-1}(\bar{\mathcal{A}}\alpha^+) - \bar\alpha^+\bar{\mathcal{A}}(\hat\partial^{-1}\partial\alpha^-)\Big)\,, \tag{A.12}$$

$$L^{--+}_{\phi A} = 2\int_\Sigma d^3\mathbf{x}\,\Big(\hat\partial\bar\phi^+(\hat\partial^{-1}\partial\bar{\mathcal{A}})\bar\phi^- - \phi^-(\hat\partial^{-1}\partial\bar{\mathcal{A}})\hat\partial\phi^+ - \phi^-\bar{\mathcal{A}}\partial\phi^+ + \partial\bar\phi^+\bar{\mathcal{A}}\bar\phi^-\Big)\,, \tag{A.13}$$

$$L^{--+}_{\alpha\Lambda\phi} = -i\sqrt{2}\int_\Sigma d^3\mathbf{x}\,\Big(\bar\phi^+(\hat\partial^{-1}\partial\bar\Lambda)\alpha^- - \bar\alpha^-(\hat\partial^{-1}\partial\bar\Lambda)\phi^+ - \bar\phi^+\bar\Lambda(\hat\partial^{-1}\partial\alpha^-) + (\hat\partial^{-1}\partial\bar\alpha^-)\bar\Lambda\phi^+\Big)\,. \tag{A.14}$$

$$L^{--++}_{A} = \int_\Sigma d^3\mathbf{x}\,\left[\frac{1}{2}\,\Sigma^a_A\hat\partial^{-2}\Sigma^a_A - \frac{1}{g^2}\mathrm{tr}\big([\mathcal{A},\bar{\mathcal{A}}]^2\big)\right]\,, \tag{A.15}$$

$$L^{--++}_{\Lambda} = \frac{1}{2}\int_\Sigma d^3\mathbf{x}\,\Sigma^a_\Lambda\hat\partial^{-2}\Sigma^a_\Lambda\,, \tag{A.16}$$

$$L^{--++}_{\Lambda A} = \int_\Sigma d^3\mathbf{x}\,\left[\Sigma^a_\Lambda\hat\partial^{-2}\Sigma^a_A - i\frac{2\sqrt{2}}{g^2}\mathrm{tr}\big([\mathcal{A},\bar\Lambda]\hat\partial^{-1}[\bar{\mathcal{A}},\Lambda]\big)\right]\,. \tag{A.17}$$

$$L^{--++}_{\phi} = \frac{1}{2}\int_\Sigma d^3\mathbf{x}\,\left[\Sigma^a_\phi\hat\partial^{-2}\Sigma^a_\phi + g^2\,(-\bar\phi^+T^a\bar\phi^- + \phi^-T^a\phi^+)^2\right]\,, \tag{A.18}$$

$$L^{--++}_{\alpha} = \frac{1}{2}\int_\Sigma d^3\mathbf{x}\,\Sigma^a_\alpha\hat\partial^{-2}\Sigma^a_\alpha\,, \tag{A.19}$$

$$L^{--++}_{\alpha\phi} = \int_\Sigma d^3\mathbf{x}\,\Big[\Sigma^a_\alpha\hat\partial^{-2}\Sigma^a_\phi - \sqrt{2}ig^2(\bar\alpha^+T^a\bar\phi^- + \phi^-T^a\alpha^+)\hat\partial^{-1}(\bar\phi^+T^a\alpha^- + \bar\alpha^-T^a\phi^+)\Big]\,. \tag{A.20}$$

$$L^{--++}_{\phi A} = \int_\Sigma d^3\mathbf{x}\,\Big[\Sigma^a_\phi\hat\partial^{-2}\Sigma^a_A + \big(\bar\phi^+\{\mathcal{A},\bar{\mathcal{A}}\}\bar\phi^- + \phi^-\{\mathcal{A},\bar{\mathcal{A}}\}\phi^+\big)\Big]\,, \tag{A.21}$$

$$L^{--++}_{\alpha\Lambda} = \int_\Sigma d^3\mathbf{x}\,\Sigma^a_\alpha\hat\partial^{-2}\Sigma^a_\Lambda\,, \tag{A.22}$$

$$L^{--++}_{\alpha A} = \int_\Sigma d^3\mathbf{x}\,\Big[\Sigma^a_\alpha\hat\partial^{-2}\Sigma^a_A - i\sqrt{2}\Big(\bar\alpha^-\mathcal{A}\hat\partial^{-1}(\bar{\mathcal{A}}\alpha^+) + \bar\alpha^+\bar{\mathcal{A}}\hat\partial^{-1}(\mathcal{A}\alpha^-)\Big)\Big]\,, \tag{A.23}$$

$$L^{--++}_{\phi\Lambda} = \int_\Sigma d^3\mathbf{x}\,\Big[\Sigma^a_\phi\hat\partial^{-2}\Sigma^a_\Lambda - i\sqrt{2}\Big(\phi^-\Lambda\hat\partial^{-1}(\bar\Lambda\phi^+) + \bar\phi^+\bar\Lambda\hat\partial^{-1}(\Lambda\bar\phi^-)\Big)\Big]\,, \tag{A.24}$$

$$
\begin{aligned}
L^{--++}_{\alpha\phi\Lambda A} = -i\sqrt{2}\int_\Sigma d^3\mathbf{x}\, \Big[ & \phi^-\Lambda\hat\partial^{-1}(\bar{\mathcal{A}}\alpha^+) - \bar\phi^+\bar\Lambda\hat\partial^{-1}(\mathcal{A}\alpha^-) - \hat\partial^{-1}(\bar\alpha^-\mathcal{A})\bar\Lambda\phi^+ \\
& + \hat\partial^{-1}(\bar\alpha^+\bar{\mathcal{A}})\Lambda\phi^- + \bar\alpha^+\big(\hat\partial^{-1}[\bar{\mathcal{A}},\Lambda]\big)\bar\phi^- - \phi^-\big(\hat\partial^{-1}[\bar{\mathcal{A}},\Lambda]\big)\alpha^+ \\
& + \bar\phi^+\big(\hat\partial^{-1}[\mathcal{A},\bar\Lambda]\big)\alpha^- - \bar\alpha^-\big(\hat\partial^{-1}[\mathcal{A},\bar\Lambda]\big)\phi^+ \Big]\,. \qquad (A.25)
\end{aligned}
$$

where

$$\Sigma^a_A = -\frac{2}{g}\mathrm{tr}\Big([\mathcal{A},\hat\partial\bar{\mathcal{A}}]T^a + [\bar{\mathcal{A}},\hat\partial\mathcal{A}]T^a\Big)\,, \qquad (A.26)$$

$$\Sigma^a_\Lambda = \frac{i2\sqrt{2}}{g}\mathrm{tr}\Big(\{\bar\Lambda,\Lambda\}T^a\Big) \qquad (A.27)$$

$$\Sigma^a_\alpha = i\sqrt{2}g\Big(\bar\alpha^- T^a\alpha^+ + \bar\alpha^+T^a\alpha^-\Big) \qquad (A.28)$$

$$\Sigma^a_\phi = -g\Big(-\hat\partial\bar\phi^+T^a\bar\phi^- + \bar\phi^+T^a\hat\partial\bar\phi^- + \phi^-T^a\hat\partial\phi^+ - \hat\partial\phi^-T^a\phi^+\Big) \qquad (A.29)$$

$$L^{+-}_{m,\alpha} = i\frac{m^2}{\sqrt{2}}\int_\Sigma d^3\mathbf{x}\,(\bar\alpha^-\hat\partial^{-1}\alpha^+ + \bar\alpha^+\hat\partial^{-1}\alpha^-)\,, \qquad (A.30)$$

$$L^{+-}_{m,\phi} = -m^2\int_\Sigma d^3\mathbf{x}\,(\bar\phi^+\bar\phi^- + \phi^-\phi^+)\,, \qquad (A.31)$$

$$L^{+-+}_{m,\alpha A} = -m\int_\Sigma d^3\mathbf{x}\;\bar\alpha^+[\hat\partial^{-1},\bar{\mathcal{A}}]\alpha^+\,, \qquad (A.32)$$

$$L^{+-+}_{m,\phi\Lambda\alpha} = m\int_\Sigma d^3\mathbf{x}\,\Big((\hat\partial^{-1}\bar\alpha^+)\bar\Lambda\phi^+ - \bar\phi^+\bar\Lambda\hat\partial^{-1}\alpha^+\Big)\,, \qquad (A.33)$$

$$L^{-+-}_{m,\alpha A} = m\int_\Sigma d^3\mathbf{x}\;\bar\alpha^-[\hat\partial^{-1},\mathcal{A}]\alpha^-\,, \qquad (A.34)$$

$$L^{-+-}_{m,\phi\Lambda\alpha} = -m\int_\Sigma d^3\mathbf{x}\,\Big(\phi^-\Lambda\hat\partial^{-1}\alpha^- - (\hat\partial^{-1}\bar\alpha^-)\Lambda\bar\phi^-\Big)\,. \qquad (A.35)$$

## B. Summary of the canonical transformation for MHV-QCD

$$\mathcal{A}^{\mathrm{QCD}}_q = \sum_{n=1}^\infty \int_{1\cdots n}\Upsilon_{q,\bar1\cdots\bar n}\mathcal{B}_1\cdots\mathcal{B}_n\,\delta_{q\bar1\cdots\bar n}, \qquad (B.1)$$

$$\alpha^{+,\mathrm{QCD}}_q = \sum_{n=1}^\infty \int_{1\cdots n}\Upsilon^+_{q,\bar1\cdots\bar n}\mathcal{B}_1\cdots\mathcal{B}_{n-1}\xi^+_n\,\delta_{q\bar1\cdots\bar n}, \qquad (B.2)$$

$$\bar\alpha^{+,\mathrm{QCD}}_q = \sum_{n=1}^\infty \int_{1\cdots n}\Xi^+_{q,\bar1\cdots\bar n}\bar\xi^+_1\mathcal{B}_2\cdots\mathcal{B}_n\,\delta_{q\bar1\cdots\bar n}, \qquad (B.3)$$

$$\alpha^{-,\mathrm{QCD}}_q = \xi^-_q + \sum_{n=2}^\infty \int_{1\cdots n}\Upsilon^-_{q,\bar1\cdots\bar n}\mathcal{B}_1\cdots\mathcal{B}_{n-1}\xi^-_n\,\delta_{q\bar1\cdots\bar n}, \qquad (B.4)$$

$$\bar\alpha^{-,\mathrm{QCD}}_q = \bar\xi^-_q + \sum_{n=2}^\infty \int_{1\cdots n}\Xi^-_{q,\bar1\cdots\bar n}\bar\xi^+_1\mathcal{B}_2\cdots\mathcal{B}_n\,\delta_{q\bar1\cdots\bar n}, \qquad (B.5)$$

$$\bar{\mathcal{A}}^{\mathrm{QCD}}_q = \bar{\mathcal{A}}^{B,\mathrm{QCD}} + \bar{\mathcal{A}}^{\xi B,\mathrm{QCD}}\,, \qquad (B.6)$$

$$\bar{\mathcal{A}}_q^{B,\mathrm{QCD}} = \frac{1}{\hat{q}} \sum_{n=1}^{\infty} \int_{1\cdots n} \sum_{s=1}^{n} \hat{s}\Xi^s_{q,\bar{1}\cdots\bar{n}} \mathcal{B}_1 \cdots \bar{\mathcal{B}}_s \cdots \mathcal{B}_n \, \delta_{q\bar{1}\cdots\bar{n}} \,, \tag{B.7}$$

$$\begin{aligned}\bar{\mathcal{A}}_q^{\xi B,\mathrm{QCD}} = \frac{g^2}{2\sqrt{2}\hat{q}} \sum_{n=2}^{\infty} \int_{1\cdots n} \Bigg[ & \sum_{s=1}^{n-1} K^{+,s}_{q,\bar{1}\cdots\bar{n}} \, \mathcal{B}_1 \cdots \mathcal{B}_{s-1} \xi_s^+ \bar{\xi}_{s+1}^- \mathcal{B}_{s+2} \cdots \mathcal{B}_n \\ & + \frac{1}{N_C} K^{+,N_C}_{q,\bar{1}\cdots\bar{n}} \, \bar{\xi}_1^- \mathcal{B}_2 \cdots \mathcal{B}_{n-1} \xi_n^+ \mathbf{I} \\ & + \sum_{s=1}^{n-1} K^{-,s}_{q,\bar{1}\cdots\bar{n}} \, \mathcal{B}_1 \cdots \mathcal{B}_{s-1} \xi_s^- \bar{\xi}_{s+1}^+ \mathcal{B}_{s+2} \cdots \mathcal{B}_n \\ & + \frac{1}{N_C} K^{-,N_C}_{q,\bar{1}\cdots\bar{n}} \, \bar{\xi}_1^+ \mathcal{B}_2 \cdots \mathcal{B}_{n-1} \xi_n^- \mathbf{I} \Bigg] \delta_{q\bar{1}\cdots\bar{n}} \,, \end{aligned} \tag{B.8}$$

where

$$\delta_{q\bar{1}\cdots\bar{n}} = (2\pi)^3 \delta^3(\vec{q} - \vec{p}_1 - \cdots \vec{p}_n) \,. \tag{B.9}$$

Define:

$$\Delta_{\bar{1}\cdots\bar{n}} = \begin{cases} \frac{\hat{1}\cdots\hat{n}}{\hat{1}\hat{n}(1\,2)\cdots(n-1,n)} \,, & \text{for } n \geq 2 \,, \\ \frac{1}{\hat{1}} \,, & \text{for } n = 1. \end{cases} \tag{B.10}$$

All the coefficients can be expressed as

$$\Upsilon_{q,\bar{1}\cdots\bar{n}} = \Upsilon^+_{q,\bar{1}\cdots\bar{n}} = \Xi^+_{q,\bar{1}\cdots\bar{n}} = (-i)^{n-1} \hat{q} \, \Delta_{\bar{1}\cdots\bar{n}} \,, \tag{B.11}$$

$$\Xi^-_{q,\bar{1}\cdots\bar{n}} = (-i)^{n-1} \hat{1} \, \Delta_{\bar{1}\cdots\bar{n}} \,, \tag{B.12}$$

$$\Upsilon^-_{q,\bar{1}\cdots\bar{n}} = (-i)^{n-1} \hat{n} \, \Delta_{\bar{1}\cdots\bar{n}} \,, \tag{B.13}$$

$$\Xi^s_{q,\bar{1}\cdots\bar{n}} = (-i)^{n-1} \hat{s} \, \Delta_{\bar{1}\cdots\bar{n}} \,, \tag{B.14}$$

$$K^{+,s}_{q,\bar{1}\cdots\bar{n}} = (-i)^{n-1} \widehat{s+1} \, \Delta_{\bar{1}\cdots\bar{n}} \,, \tag{B.15}$$

$$K^{+,N_C}_{q,\bar{1}\cdots\bar{n}} = -(-i)^{n-1} \hat{1} \, \Delta_{\bar{1}\cdots\bar{n}} \,, \tag{B.16}$$

$$K^{-,s}_{q,\bar{1}\cdots\bar{n}} = -(-i)^{n-1} \hat{s} \, \Delta_{\bar{1}\cdots\bar{n}} \,, \tag{B.17}$$

$$K^{-,N_C}_{q,\bar{1}\cdots\bar{n}} = (-i)^{n-1} \hat{n} \, \Delta_{\bar{1}\cdots\bar{n}} \,. \tag{B.18}$$

## C. Summary of the canonical transformation for MHV-SQCD

$$\mathcal{A}_q = \mathcal{A}_q^{\mathrm{QCD}} \,, \quad \alpha_q^+ = \alpha_q^{+,\mathrm{QCD}} \,, \quad \bar{\alpha}_q^+ = \bar{\alpha}_q^{+,\mathrm{QCD}} \,, \tag{C.1}$$

$$\alpha_q^- = \alpha_q^{-,\mathrm{QCD}} - \sum_{n=2}^{\infty} \int_{1\cdots n} \Upsilon^-_{q,\bar{1}\cdots\bar{n}} \left( \sum_{l=1}^{n-1} \mathcal{B}_1 \cdots \Pi_l \cdots \mathcal{B}_{n-1} \bar{\varphi}_n^- \right) \delta_{q\bar{1}\cdots\bar{n}}, \tag{C.2}$$

$$\bar{\alpha}_q^- = \bar{\alpha}_q^{-,\mathrm{QCD}} + \sum_{n=2}^{\infty} \int_{1\cdots n} \Xi^-_{q,\bar{1}\cdots\bar{n}} \left( \sum_{l=2}^{n} \varphi_1^- \mathcal{B}_2 \cdots \Pi_l \cdots \mathcal{B}_n \right) \delta_{q\bar{1}\cdots\bar{n}}, \tag{C.3}$$

$$\Lambda_q = \sum_{n=1}^{\infty} \int_{1\cdots n} \Upsilon_{q,\bar{1}\cdots\bar{n}} \sum_{l=1}^{n} \mathcal{B}_1 \cdots \Pi_l \cdots \mathcal{B}_n \, \delta_{q\bar{1}\cdots\bar{n}}, \tag{C.4}$$

$$\phi_q^+ = \varphi_q^+ + \frac{1}{\hat{q}} \sum_{n=2}^{\infty} \int_{1\cdots n} \Upsilon^+_{q,\bar{1}\cdots\bar{n}} \left( \hat{n} \mathcal{B}_1 \cdots \mathcal{B}_{n-1} \varphi_n^+ - \frac{1}{\sqrt{2}} \sum_{l=1}^{n-1} \mathcal{B}_1 \cdots \Pi_l \cdots \mathcal{B}_{n-1} \xi_n^+ \right) \delta_{q\bar{1}\cdots\bar{n}}, \tag{C.5}$$

$$\bar{\phi}_q^+ = \bar{\varphi}_q^+ + \frac{1}{\hat{q}} \sum_{n=2}^{\infty} \int_{1\cdots n} \Xi^+_{q,\bar{1}\cdots\bar{n}} \left( \hat{1} \bar{\varphi}_1^+ \mathcal{B}_2 \cdots \mathcal{B}_n - \frac{1}{\sqrt{2}} \sum_{l=2}^{n} \bar{\xi}_1^+ \mathcal{B}_2 \cdots \Pi_l \cdots \mathcal{B}_n \right) \delta_{q\bar{1}\cdots\bar{n}}, \tag{C.6}$$

$$\bar{\phi}_q^- = \sum_{n=1}^{\infty} \int_{1\cdots n} \Upsilon^-_{q,\bar{1}\cdots\bar{n}} \mathcal{B}_1 \cdots \mathcal{B}_{n-1} \bar{\varphi}_n^- \, \delta_{q\bar{1}\cdots\bar{n}}, \tag{C.7}$$

$$\phi_q^- = \sum_{n=1}^{\infty} \int_{1\cdots n} \Xi^-_{q,\bar{1}\cdots\bar{n}} \varphi_1^- \mathcal{B}_2 \cdots \mathcal{B}_n \, \delta_{q\bar{1}\cdots\bar{n}}, \tag{C.8}$$

$$\bar{\Lambda}_q = \bar{\Lambda}_q^{B\Pi} + \bar{\Lambda}_q^{\phi\xi B}, \tag{C.9}$$

$$\bar{\Lambda}_q^{B\Pi} = \sum_{n=1}^{\infty} \int_{1\cdots n} \sum_{l=1}^{n} \Xi^l_{q,\bar{1}\cdots\bar{n}} \mathcal{B}_1 \cdots \bar{\Pi}_l \cdots \mathcal{B}_n \, \delta_{q\bar{1}\cdots\bar{n}}, \tag{C.10}$$

$$\begin{aligned} \bar{\Lambda}_q^{\varphi\xi B} = \frac{g^2}{2} \sum_{n=2}^{\infty} \int_{1\cdots n} \Bigg[ & \sum_{s=1}^{n-1} K^{+,s}_{q,\bar{1}\cdots\bar{n}} \mathcal{B}_1 \cdots \mathcal{B}_{s-1} \xi_s^+ \varphi_{s+1}^- \mathcal{B}_{s+2} \cdots \mathcal{B}_n \\ & - \frac{1}{N_C} K^{+,N_C}_{q,\bar{1}\cdots\bar{n}} \varphi_1^- \mathcal{B}_2 \cdots \mathcal{B}_{n-1} \xi_n^+ \mathbf{I} \\ & + \sum_{s=1}^{n-1} K^{-,s}_{q,\bar{1}\cdots\bar{n}} \mathcal{B}_1 \cdots \mathcal{B}_{s-1} \bar{\varphi}_s^- \bar{\xi}_{s+1}^+ \mathcal{B}_{s+2} \cdots \mathcal{B}_n \\ & - \frac{1}{N_C} K^{-,N_C}_{q,\bar{1}\cdots\bar{n}} \bar{\xi}_1^+ \mathcal{B}_2 \cdots \mathcal{B}_{n-1} \bar{\varphi}_n^- \mathbf{I} \Bigg] \delta_{q\bar{1}\cdots\bar{n}} . \end{aligned} \tag{C.11}$$

$$\bar{\mathcal{A}}_q = \bar{\mathcal{A}}^B + \bar{\mathcal{A}}^{B\Pi} + \bar{\mathcal{A}}^{\xi B} + \bar{\mathcal{A}}^{\varphi B} + \bar{\mathcal{A}}^{\varphi\xi\Pi B}, \tag{C.12}$$

$$\bar{\mathcal{A}}_q^B = \bar{\mathcal{A}}_q^{B,\mathrm{QCD}}, \quad \bar{\mathcal{A}}_q^{\xi B} = \bar{\mathcal{A}}_q^{\xi B,\mathrm{QCD}}, \tag{C.13}$$

$$\bar{\mathcal{A}}_q^{B\Pi} = -\frac{1}{\sqrt{2}\hat{q}} \sum_{n=2}^{\infty} \int_{1\cdots n} \sum_{s=1}^{n} \left[ \Xi^s_{q,\bar{1}\cdots\bar{n}} \sum_{l=1, l\neq s}^{n} (-1)^{\delta_{ls}} \mathcal{B}_1 \cdots \Pi_l \cdots \bar{\Pi}_s \cdots \mathcal{B}_n \right] \delta_{q\bar{1}\cdots\bar{n}}, \tag{C.14}$$

$$\begin{aligned} \bar{\mathcal{A}}_q^{\varphi B} = \frac{g^2}{2\hat{q}} \sum_{n=2}^{\infty} \int_{1\cdots n} \Bigg[ & \sum_{s=1}^{n-1} \hat{s} K^{+,s}_{q,\bar{1}\cdots\bar{n}} \mathcal{B}_1 \cdots \mathcal{B}_{s-1} \varphi_s^+ \varphi_{s+1}^- \mathcal{B}_{s+2} \cdots \mathcal{B}_n \\ & - \frac{\hat{n}}{N_C} K^{+,N_C}_{q,\bar{1}\cdots\bar{n}} \varphi_1^- \mathcal{B}_2 \cdots \mathcal{B}_{n-1} \varphi_n^+ \mathbf{I} \\ & - \sum_{s=1}^{n-1} \widehat{s+1}\, K^{-,s}_{q,\bar{1}\cdots\bar{n}} \mathcal{B}_1 \cdots \mathcal{B}_{s-1} \bar{\varphi}_s^- \bar{\varphi}_{s+1}^+ \mathcal{B}_{s+2} \cdots \mathcal{B}_n \\ & + \frac{\hat{1}}{N_C} K^{-,N_C}_{q,\bar{1}\cdots\bar{n}} \bar{\varphi}_1^+ \mathcal{B}_2 \cdots \mathcal{B}_{n-1} \bar{\varphi}_n^- \mathbf{I} \Bigg] \delta_{q\bar{1}\cdots\bar{n}}, \end{aligned} \tag{C.15}$$

$$\begin{aligned} \bar{\mathcal{A}}_q^{\varphi\xi\Pi B} = \frac{g^2}{2\sqrt{2}\hat{q}} \sum_{n=3}^{\infty} \int_{1\cdots n} \Bigg[ & - \sum_{s=1}^{n-1} K^{+,s}_{q,\bar{1}\cdots\bar{n}} \left( \sum_{l=1, l\neq s,s+1}^{n} (-1)^{\delta_{ls}} \mathcal{B}_1 \cdots \Pi_l \cdots \xi_s^+ \varphi_{s+1}^- \cdots \mathcal{B}_n \right) \\ & + \frac{1}{N_C} K^{+,N_C}_{q,\bar{1}\cdots\bar{n}} \sum_{s=2}^{n-1} \varphi_1^- \mathcal{B}_2 \cdots \Pi_s \cdots \mathcal{B}_{n-1} \xi_n^+ \mathbf{I} \end{aligned}$$

$$
\begin{aligned}
&-\sum_{s=1}^{n-1} K_{q,\bar{1}\cdots\bar{n}}^{-,s}\left(\sum_{l=1,l\neq s,s+1}^{n}(-1)^{\delta_{ls}}\mathcal{B}_1\cdots\Pi_l\cdots\bar{\varphi}_s^-\bar{\xi}_{s+1}^+\mathcal{B}_{s+2}\cdots\mathcal{B}_n\right)\\
&-\frac{1}{N_C}K_{q,\bar{1}\cdots\bar{n}}^{-,N_C}\sum_{s=2}^{n-1}\bar{\xi}_1^+\mathcal{B}_2\cdots\Pi_s\cdots\mathcal{B}_{n-1}\bar{\varphi}_n^-\mathbf{I}\Bigg]\delta_{q\bar{1}\cdots\bar{n}}\,, \qquad \text{(C.16)}
\end{aligned}
$$

where $\mathbf{I}$ is the color singlet unit matrix and

$$
\delta_{ls} = \begin{cases} 0 & \text{for } l < s\,, \\ 1 & \text{for } l > s\,. \end{cases} \qquad \text{(C.17)}
$$

## D. Summary of the massive CSW vertices for MHV-QCD

$$
\begin{aligned}
L_m^{+-} = \frac{m^2}{\sqrt{2}}\Bigg[&\sum_{n=2}^{\infty}\int_{1,2,\cdots n} V_m^{-+}(\bar{1}\cdots\bar{n})\bar{\xi}_1^-\mathcal{B}_2\cdots\mathcal{B}_{n-1}\xi_n^+\,\delta_{1\cdots n}\\
&+\sum_{n=2}^{\infty}\int_{1,2,\cdots n} V_m^{+-}(\bar{1}\cdots\bar{n})\bar{\xi}_1^+\mathcal{B}_2\cdots\mathcal{B}_{n-1}\xi_n^-\,\delta_{1\cdots n}\Bigg]\,, \qquad \text{(D.1)}
\end{aligned}
$$

where

$$
\delta_{1\ldots n} = (2\pi)^3\delta^3\Big(\sum_{i=1}^{n} p_i\Big)\,, \qquad \text{(D.2)}
$$

$$
V_m^{-+}(\bar{1}\cdots\bar{n}) = (-i)^{n-2}\Delta_{\bar{1}\cdots\bar{n}}\frac{(1\,n)}{\hat{n}}\,, \qquad \text{(D.3)}
$$

$$
V_m^{+-}(\bar{1}\cdots\bar{n}) = -(-i)^{n-2}\Delta_{\bar{1}\cdots\bar{n}}\frac{(1\,n)}{\hat{1}}\,, \qquad \text{(D.4)}
$$

and $\Delta$ is defined in (B.10).

$$
\begin{aligned}
L_{Fm}^{+-+} = im\Bigg[&\sum_{n=3}^{\infty}\sum_{s=2}^{n-1}\int_{1\cdots n} V^{s,+-+}(\bar{1}\cdots\bar{n})\bar{\xi}_1^+\mathcal{B}_2\cdots\bar{\mathcal{B}}_s\cdots\mathcal{B}_{n-1}\xi_n^+\,\delta_{1\cdots n}\\
&+\frac{g^2}{2\sqrt{2}}\sum_{n=4}^{\infty}\sum_{s=2}^{n-2}\int_{1,2,\cdots n}\Big(V^{s,++-+}(\bar{1}\cdots\bar{n})\bar{\xi}_1^+\mathcal{B}_2\cdots\mathcal{B}_{s-1}\xi_s^+\bar{\xi}_{s+1}^-\mathcal{B}_{s+2}\cdots\mathcal{B}_{n-1}\xi_n^+\\
&+V^{s,+-++}(\bar{1}\cdots\bar{n})\bar{\xi}_1^+\mathcal{B}_2\cdots\mathcal{B}_{s-1}\xi_s^-\bar{\xi}_{s+1}^+\mathcal{B}_{s+2}\cdots\mathcal{B}_{n-1}\xi_n^+\Big)\delta_{1\cdots n}\Bigg]\,, \qquad \text{(D.5)}
\end{aligned}
$$

where

$$
\begin{aligned}
V^{s,+-+}(\bar{1}\cdots\bar{n}) &= (-i)^{n-3}\Delta_{\bar{1}\cdots\bar{n}}\frac{(1\,s)(s\,n)}{\hat{1}\hat{n}}\,, && \text{for } n\geq 3\,,\\
V^{s,++-+}(\bar{1}\cdots\bar{n}) &= (-i)^{n-3}\Delta_{\bar{1}\cdots\bar{n}}\frac{(1\,s)}{\hat{1}\hat{s}}\left(\frac{(s+1\,n)}{\hat{n}}-\frac{1}{N_C}\frac{(s\,s+1)}{\hat{s}}\right)\,, && \text{for } n\geq 4\,,\\
V^{s,+-++}(\bar{1}\cdots\bar{n}) &= -(-i)^{n-3}\Delta_{\bar{1}\cdots\bar{n}}\frac{(s+1\,n)}{\widehat{s+1}\hat{n}}\left(\frac{(1\,s)}{\hat{1}}-\frac{1}{N_C}\frac{(s\,s+1)}{\widehat{s+1}}\right)\,, && \text{for } n\geq 4\,.
\end{aligned}
\qquad \text{(D.6)}
$$

$$L_{Fm}^{-+-} = im \sum_{n=3}^{\infty} \int_{1,2,\cdots n} V^{-+-}(\bar{1} \cdots \bar{n}) \bar{\xi}_1^- \mathcal{B}_2 \mathcal{B}_3 \cdots \mathcal{B}_{n-1} \xi_n^- \, \delta_{1 \cdots n} \,, \tag{D.7}$$

where

$$V^{-+-}(\bar{1} \cdots \bar{n}) = -(-i)^{n-3} \Delta_{\bar{1} \cdots \bar{n}} \frac{(1\, n)^2}{\hat{1} \hat{n}} \,, \quad \text{for } n \geq 3 \,. \tag{D.8}$$

## References


[1] S.J. Parke and T.R. Taylor, *An amplitude for n gluon scattering*, *Phys. Rev. Lett.* **56** (1986) 2459.

[2] F.A. Berends and W.T. Giele, *Multiple soft gluon radiation in parton processes*, *Nucl. Phys.* **B 313** (1989) 595.

[3] F. Cachazo, P. Svrček and E. Witten, *MHV vertices and tree amplitudes in gauge theory*, *JHEP* **09** (2004) 006 [hep-th/0403047].

[4] J.-B. Wu and C.-J. Zhu, *MHV vertices and scattering amplitudes in gauge theory*, *JHEP* **07** (2004) 032 [hep-th/0406085].

[5] J.-B. Wu and C.-J. Zhu, *MHV vertices and fermionic scattering amplitudes in gauge theory with quarks and gluinos*, *JHEP* **09** (2004) 063 [hep-th/0406146].

[6] G. Georgiou and V.V. Khoze, *Tree amplitudes in gauge theory as scalar MHV diagrams*, *JHEP* **05** (2004) 070 [hep-th/0404072].

[7] G. Georgiou, E.W.N. Glover and V.V. Khoze, *Non-MHV tree amplitudes in gauge theory*, *JHEP* **07** (2004) 048 [hep-th/0407027].

[8] L.J. Dixon, E.W.N. Glover and V.V. Khoze, *MHV rules for Higgs plus multi-gluon amplitudes*, *JHEP* **12** (2004) 015 [hep-th/0411092].

[9] S.D. Badger, E.W.N. Glover and V.V. Khoze, *MHV rules for Higgs plus multi-parton amplitudes*, *JHEP* **03** (2005) 023 [hep-th/0412275].

[10] Z. Bern, D. Forde, D.A. Kosower and P. Mastrolia, *Twistor-inspired construction of electroweak vector boson currents*, *Phys. Rev.* **D 72** (2005) 025006 [hep-ph/0412167].

[11] A. Brandhuber, B.J. Spence and G. Travaglini, *One-loop gauge theory amplitudes in $N = 4$ super Yang-Mills from MHV vertices*, *Nucl. Phys.* **B 706** (2005) 150 [hep-th/0407214].

[12] J. Bedford, A. Brandhuber, B.J. Spence and G. Travaglini, *A twistor approach to one-loop amplitudes in $N = 1$ supersymmetric Yang-Mills theory*, *Nucl. Phys.* **B 706** (2005) 100 [hep-th/0410280].

[13] J. Bedford, A. Brandhuber, B.J. Spence and G. Travaglini, *Non-supersymmetric loop amplitudes and MHV vertices*, *Nucl. Phys.* **B 712** (2005) 59 [hep-th/0412108].

[14] A. Brandhuber, B. Spence and G. Travaglini, *From trees to loops and back*, *JHEP* **01** (2006) 142 [hep-th/0510253].

[15] C. Quigley and M. Rozali, *One-loop MHV amplitudes in supersymmetric gauge theories*, *JHEP* **01** (2005) 053 [hep-th/0410278].

[16] A. Brandhuber, B. Spence and G. Travaglini, *Amplitudes in pure Yang-Mills and MHV diagrams*, *JHEP* **02** (2007) 088 [hep-th/0612007].




JHEP12(2008)028

[17] A. Brandhuber, B. Spence, G. Travaglini and K. Zoubos, *One-loop MHV rules and pure Yang-Mills*, *JHEP* **07** (2007) 002 [arXiv:0704.0245].

[18] R. Britto, F. Cachazo, B. Feng and E. Witten, *Direct proof of tree-level recursion relation in Yang-Mills theory*, *Phys. Rev. Lett.* **94** (2005) 181602 [hep-th/0501052].

[19] K. Risager, *A direct proof of the CSW rules*, *JHEP* **12** (2005) 003 [hep-th/0508206].

[20] R. Britto, F. Cachazo and B. Feng, *New recursion relations for tree amplitudes of gluons*, *Nucl. Phys.* **B 715** (2005) 499 [hep-th/0412308].

[21] A. Gorsky and A. Rosly, *From Yang-Mills Lagrangian to MHV diagrams*, *JHEP* **01** (2006) 101 [hep-th/0510111].

[22] P. Mansfield, *The Lagrangian origin of MHV rules*, *JHEP* **03** (2006) 037 [hep-th/0511264].

[23] J.H. Ettle and T.R. Morris, *Structure of the MHV-rules Lagrangian*, *JHEP* **08** (2006) 003 [hep-th/0605121].

[24] J.H. Ettle, C.-H. Fu, J.P. Fudger, P.R.W. Mansfield and T.R. Morris, *S-matrix equivalence theorem evasion and dimensional regularisation with the canonical MHV Lagrangian*, *JHEP* **05** (2007) 011 [hep-th/0703286].

[25] R. Boels, L. Mason and D. Skinner, *From twistor actions to MHV diagrams*, *Phys. Lett.* **B 648** (2007) 90 [hep-th/0702035].

[26] R. Boels, L. Mason and D. Skinner, *Supersymmetric gauge theories in twistor space*, *JHEP* **02** (2007) 014 [hep-th/0604040].

[27] R. Boels and C. Schwinn, *CSW rules for a massive scalar*, *Phys. Lett.* **B 662** (2008) 80 [arXiv:0712.3409].

[28] R. Boels and C. Schwinn, *Deriving CSW rules for massive scalar legs and pure Yang-Mills loops*, *JHEP* **07** (2008) 007 [arXiv:0805.1197].

[29] J.H. Ettle, T.R. Morris and Z. Xiao, *The MHV QCD Lagrangian*, *JHEP* **08** (2008) 103 [arXiv:0805.0239].

[30] R. Boels and C. Schwinn, *CSW rules for massive matter legs and glue loops*, *Nucl. Phys.* **183** *(Proc. Suppl.)* (2008) 137 [arXiv:0805.4577].

[31] C. Schwinn, *Twistor-inspired construction of massive quark amplitudes*, *Phys. Rev.* **D 78** (2008) 085030 [arXiv:0809.1442].

[32] C. Schwinn and S. Weinzierl, *SUSY Ward identities for multi-gluon helicity amplitudes with massive quarks*, *JHEP* **03** (2006) 030 [hep-th/0602012].

[33] H. Feng and Y.-T. Huang, *MHV Lagrangian for $N = 4$ super Yang-Mills*, hep-th/0611164.